How optics resolved the Einstein-Podolsky-Rosen paradox

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Einstein-Bohr debate

Quantum mechanics:

- Wave-particle duality
- Hilbert space
- Quantum superposition
- Heisenberg uncertainty

\[ \Delta x \Delta p \geq \frac{\hbar}{2} \]

Albert Einstein and Neils Bohr debating quantum mechanics
Quantum mechanics: measurements on one particle dictate the state of the other particle.
Bell’s inequality

Assumptions:
- Only local interactions
  (no "spooky action at a distance")
- Unique reality
  (not multiple realities)

Then:

\[ B(a, b, a', b') = |C(a, b) - C(a, b')| + |C(a', b) + C(a', b')| \leq 2 \]

where \( C \) is the correlation between two measurements:

\[ C(a, b) = E \{ A(a) B(b) \} \]
4 possibilities

- **Local**
  - Unique reality: Hidden variables (Classical theories)
  - Multiple realities: Copenhagen interpretation

- **Non-local**
  - Unique reality: Alternative interpretation
  - Multiple realities: Undesirable
Aspect experiment
Type II phase matching \( \Rightarrow \) photons have perpendicular polarization: \( \theta_B = \theta_A - \pi/2 \)

However, each beam is unpolarized — contains all states of polarization.
Before considering the calculations, we first need to review dichroic polarizers.

Intensity: \( I_{\text{ver}} = I_{\text{in}} \cos^2(\theta) \)

where \( \theta \) is the angle between vertical polarization and the input polarization.

Probability to detect photon: \( P(\theta) = \cos^2(\theta) \)
Classical calculations

Orientation angles of the two polarizers: $\alpha_A$ and $\alpha_B$

Probability to detect photons at respective detectors:

$$P(A|\theta_A, \alpha_A) = \cos^2(\theta_A - \alpha_A)$$

$$P(B|\theta_B, \alpha_B) = \cos^2(\theta_B - \alpha_B) = \sin^2(\theta_A - \alpha_B)$$

Detectors are far apart $\rightarrow$ assume the probabilities of detecting photons are statistically independent

$$\Rightarrow P(AB|\theta_A, \alpha_A, \alpha_B) = P(A|\theta_A, \alpha_A)P(B|\theta_A, \alpha_B)$$

$$= \frac{1}{4} [\sin(\alpha_A - \alpha_B) + \sin(2\theta_A - \alpha_A - \alpha_B)]^2$$
Classical case

For the same orientation:
angle: \((\alpha_A = \alpha_B = \alpha)\):

\[
P(AB|\alpha, \alpha) = \frac{1}{8}
\]

Integrate over all polarization states:

\[
P(AB|\alpha_A, \alpha_B) = \frac{1}{2\pi} \int_{0}^{2\pi} P(AB|\theta_A, \alpha_A, \alpha_B) \, d\theta_A
\]

\[
= \frac{1}{8} + \frac{1}{4} \sin^2(\alpha_A - \alpha_B)
\]
Quantum calculations

2-state maximally entangled bi-partite (2 photon) system:

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|\text{hor, ver}\rangle - |\text{ver, hor}\rangle) \]

\[ a_n = a_{n,\text{hor}} \cos \alpha_n + a_{n,\text{ver}} \sin \alpha_n \quad n = A, B \]

\[ a_n^\dagger = a_{n,\text{hor}}^\dagger \cos \alpha_n + a_{n,\text{ver}}^\dagger \sin \alpha_n \quad n = A, B \]

\[ P(A, B|\alpha_A, \alpha_B) = \langle \psi| a_A^\dagger a_B^\dagger a_A a_B |\psi\rangle = \frac{1}{2} \sin^2 (\alpha_A - \alpha_B) \]

So for \( \alpha_A - \alpha_B = 0 \) (same orientation) \( P(A, B|\alpha, \alpha) = 0 \)
Comparison

P(A,B|0,α)

Classical

P(A,B|0,α)

Quantum mechanical
Experimental results

For polarization ...

\[ R(\phi)/R_0 \]

\[
\begin{array}{cccc}
0 & 90 & 180 & 270 & 360 \\
0.5 & 0.4 & 0.3 & 0.2 & 0.1 \\
\end{array}
\]

\( (\text{DEGREES}) \)

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More experimental results

(... and for orbital angular momentum)

\[ a \]

\[ J. \text{ Leach, et al., Optics Express, Vol 17, p. 8287 (2009).} \]
Test of Bell’s inequality

Theoretically:
Classical case: \( C_C(\alpha_A, \alpha_B) = -\frac{1}{2} \cos(2\alpha_A - 2\alpha_B) \)
Quantum case: \( C_Q(\alpha_A, \alpha_B) = -\cos(2\alpha_A - 2\alpha_B) \)

Maximum violation for:
\[ \alpha_A = 0, \alpha_B = \frac{3\pi}{8}, \alpha_A' = -\frac{\pi}{4}, \alpha_B' = \frac{\pi}{8} \]

Gives
\[ B_C(\alpha_A, \alpha_B, \alpha_A', \alpha_B') = 1.4 < 2 \]
and
\[ B_Q(\alpha_A, \alpha_B, \alpha_A', \alpha_B') = 2.8 > 2 \]

Experimentally Bell’s (or related) inequality was violated by several standard deviations.
How does it work?

Nonlinear crystal

Quantum collapse
<table>
<thead>
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Where can we use it?

▷ Quantum communication — quantum cryptography
▷ Quantum computing — efficient factorization
▷ Ghost imaging
▷ Exotic photon sources