Intra-cavity metamorphosis of a Gaussian beam to flat-top distribution

Darryl Naidooa,b†, Igor Litvina, Alexander Laskinc and Andrew Forbesa,b

aCSIR National Laser Centre, P. O. Box 395, Pretoria 0001, South Africa
bLaser Research Institute, University of Stellenbosch, Stellenbosch 7602, South Africa
cAdlOptica GmbH, Rudower Chaussee 29, 12489 Berlin, Germany

ABSTRACT

We explore an intra-cavity beam shaping approach to generate a Gaussian distribution by the metamorphosis of a Gaussian beam into a flat-top distribution on opposing mirrors. The concept is tested external to the cavity through the use of two spatial light modulators (SLM), where the first SLM is used to transform a collimated Gaussian into a flat-top distribution and the second SLM is encoded with the conjugate phase of the flat-top for conversion back to a Gaussian. We implement this intra-cavity selection through the use of two optical elements of the refractive variant that are designed from the phase profiles addressed to the SLMs. We consider a solid-state diode side-pumped laser resonator that consists of two planar mirrors where the refractive optics are positioned at the mirrors. We out couple the Gaussian and show that the output beam size is comparable with the theoretical predictions and that we have an increase in optical brightness when compared to the cavity without any optics.

Keywords: Gaussian beam, Flat-top beam, Optical brightness.

1. INTRODUCTION

High power solid-state lasers that emit a beam with an intensity profile that is as plane as possible, are of considerable interest in industry, the military and academia, and have been exploited in applications such as remote sensing, ultrafast spectroscopy, laser precision materials processing and laser weapons applications. There are in principle two methods in which to generate a flat-top beam (FTB), namely intra- and extra-cavity beam shaping. The latter has been extensively reviewed to date where an output beam from a laser is manipulated by suitably choosing an amplitude and/or phase mask. Intra-cavity beam shaping techniques allow for the FTB to be generated directly as the cavity output mode and are more desirable than extra-cavity techniques as this ensures that the gain is coupled into the desired mode and diffraction effects and other unavoidable losses are reduced. There are several advantages to executing intra-cavity beam shaping for FTBs which include controlling the output size with conventional imaging systems without the need for specialised optics and the potential for higher energy extraction due to a larger mode volume. The primary disadvantage is that FTBs are mathematically not eigen solutions to laser resonators with spherical curvature mirrors and thus cannot be generated from conventional Fabry-Perot type resonators.

Typically, the selection of a low-loss fundamental mode may be achieved through a suitable choice of an internal aperture in a stable laser resonator. This, however, results in poor energy extraction due to the small beam waist and poor power loss discrimination between low-order modes. A higher energy extraction is achievable in an unstable laser resonator through a large mode volume but at the expense of high intrinsic losses for the oscillating modes. These unfavourable characteristics are overcome by the introduction of a low loss stable graded-phase mirror (GPM) which when implemented in a resonator presents a very large discrimination of higher-order modes by altering the generalised radius of curvature of some incoming beam. This principle has been successfully implemented to solid-state lasers and has been furthered by the inclusion of an intra-cavity phase plate for improving the discrimination. There is, however, a limitation with the use of such a GPM in that when the mirror is spherical, the change in curvature of the beam is not dependent on the amplitude of the field incident on the mirror. This implies that the GPM is not suitable for the selection of Hermite-Gaussian or Laguerre-Gaussian modes in the resonator which includes the most fundamental of these families of modes, the Gaussian mode.

† Corresponding author: Darryl Naidoo; tel: +27 12 841 3797; fax: +27 12 841 3152; email: dnaidoo3@csir.co.za
This deficiency is overcome by metamorphosing a Gaussian beam into some other desired shape, such as a FTB\textsuperscript{8,9}. In this paper we explore a laser resonator with the use of two optical elements of the refractive variant to select a Gaussian beam by the intra-cavity metamorphosis of a Gaussian beam at the output coupler to a FTB on the opposing mirror. This technique is first tested external to the cavity by the use of two spatial light modulators (SLM) to mimic the unfolded cavity, where the phase profiles of the two optical elements are addressed to the respective SLMs. These phase profiles are used to design physical optical elements for intra-cavity use where we show an increase in the brightness of the output as compared to an unstable cavity.

2. PHASE PROFILES

In this section we outline a functional form of the required phase profiles of the optical elements. Each element is positioned at a mirror where at the output coupler ($M_1$, see Fig. 1) we consider a Gaussian field of the form $u_G(\rho) = \exp\left\{-\frac{\rho^2}{w_0^2}\right\}$, where $w_0$ is the Gaussian beam width. If the element at $M_1$ is comprised of a Fourier transforming lens and some phase-only transmission component, $\psi_F$, where the optical length of the cavity matches that of the Fourier transforming lens ($L = f$), then the resulting field at the opposing mirror $M_2$ may be expressed as:\textsuperscript{9}

$$u_F = -i \frac{k}{f} \exp\left\{ikf + \frac{ikr^2}{2f}\right\} \int_0^{\infty} u_G(\rho) \exp[i\psi_F(\rho)]J_0\left(\frac{kr}{f}\right) \rho d\rho.$$  \hspace{1cm} (1)

We may determine an analytical solution for the phase function of the phase-only transmission component through the stationary phase approximation such that $u_F$ is a flat-top beam of width $w_{FTB}$ by\textsuperscript{10}:

$$\psi_F(\rho) = \alpha \frac{\sqrt{\pi}}{2} \int_0^{\rho_\text{FTB}} \sqrt{1 - \exp(-\zeta^2)} d\zeta,$$ \hspace{1cm} (2)

where $\alpha$ is a dimensionless parameter that is defined as:

$$\alpha = \frac{2\pi w_{FTB}}{f\lambda}.$$ \hspace{1cm} (3)

The flat-top beam is generated on $M_2$ which is the Fourier plane of the lens and the phase profile of the element at $M_1$ has a dual function of the lens and element which is given as:

$$\psi_1(\rho) = \psi_F(\rho) - \frac{k\rho^2}{2f},$$ \hspace{1cm} (4)

where the term $k\rho^2/2f$ is required for the Fourier lens. As with the phase profile of the first element, $\psi_1$, it is also possible to apply the stationary phase approximation to determine a closed form solution for the phase profile of the element at $M_2$ as:

$$\psi_2(r) = \arg\left\{\exp\left[i\frac{kr^2}{2f} + \psi_F(\rho(r)) - \frac{\alpha r^2(r)}{w_0 w_{FTB}}\right]\right\},$$ \hspace{1cm} (5)

where the unknown function $\rho(r)$ may be determined from the stationary phase condition $r/w_{FTB} = \partial \psi_F/\partial \rho$ with:
\[
\rho(r) = W_0 \sqrt{-\ln \left[1 - \left(\frac{2r}{\sqrt{\pi} w_{FTB}}\right)^2\right]}. \quad (6)
\]

Such an element in combination with \( M_2 \) will reproduce a Gaussian field at \( M_1 \) with a flat wavefront as desired.

3. EXTRA-CAVITY VERIFICATION

The concept of the laser resonator as presented in Fig. 1 was tested external to the cavity through a digital approach. We implement a phase profile, \( \psi_1 \), onto a spatial light modulator (SLM1) to perform the transformation of a Gaussian beam into a circular FTB. A second SLM (SLM2) is then used which is encoded with an appropriate phase function, \( \psi_2 \), to perform the reverse propagation of the FTB into a Gaussian beam. Experimentally, we propagated a collimated Gaussian beam of \( 4\sigma = 3.57 \) mm onto SLM1 which was addressed with the appropriate phase function multiplied to a Fourier lens and at the focal length of the lens (\( f = 500 \) mm, Fourier plane) the flat-top was realised at plane 1 as in Fig. 2. At this plane we positioned SLM2 to perform the reverse propagation of the FTB into a Gaussian beam. We tested for two FTBs, 2 mm in diameter corresponding to \( \alpha = 25 \) and 4 mm in diameter corresponding to \( \alpha = 50 \), where the phase functions addressed to SLM2 were adjusted accordingly to the difference in FTB size.

The intensity profiles of the collimated Gaussian beam (Fig. 3(a) and (d)) at the plane of SLM1, circular flat-top beams at plane 1, of 2 mm diameter (Fig. 3(b)) and 4 mm diameter (Fig. 3(e)), and their respective reverse propagated beams (Fig. 3(c) and (f)) are recorded on a CCD camera (Spiricon Beamgage). As a measure of the quality of the initial Gaussian beam and the reverse propagated beams we considered an ISO parameter, namely, Roughness of fit (Ro),
which is a measure of the maximum deviation of the theoretical fit to the measured distribution. This value lies between 0 and 1 and where 0 is a perfect Gaussian fit. The Gaussian beams in Fig. 3(a) and (d) have $R_o = 0.189$ with Fig. 3(c) $R_o = 0.360$ and Fig. 3(f) $R_o = 0.277$. The reverse propagated outputs are Gaussian in shape, however, with a fair degree of noise. The noise is due to imperfect transformation and can be corrected with high resolution images of the phase on the SLM and accurate sizing of the phase pattern corresponding to the incident flat-top size. This noise, however, will be accounted for within the laser resonator and we anticipate a smooth output profile.

Figure 3: Intensity profiles of the (a) and (d) incident Gaussian beam, that are used to select a (b) FTB of 2 mm diameter with its reverse propagated (c) Gaussian beam. Similarly, we select a (e) FTB of 4 mm diameter with its reverse propagated (f) Gaussian beam.

4. INTRA-CAVITY GAUSSIAN METAMORPHOSIS

The external test provide strong affirmation that the technique through the use of two SLMs presents well defined FTBs which are reverse propagated to Gaussian beams of the similar diameter to that of the input Gaussian beams. Optical elements of the refractive variant are opted for the intra-cavity selection of the Gaussian beam where the phase profiles of the respective optics are similar to that in the extra-cavity test. We implemented the optics into a diode side-pumped solid-state resonator and positioned the optics as close to the mirrors as possible. The laser resonator consists of an active medium, Nd:YAG rod, positioned close to the second element of phase profile $\psi_2$, a flat back mirror and a flat output coupler where the first element of phase profile $\psi_1$, is positioned as is illustrated in the experimental schematic in Fig. 4.

Figure 4: Experimental setup of an unstable (flat-flat) cavity that consists of the optics inserted at the mirrors.

The key point of comparison in this study is that of the system presented in Fig. 4 against an optical resonator of the same length and output mirrors, however, without the inclusion of the optics termed an empty cavity. The output profiles of the laser operating without the optics are presented in Fig. 5(a) and (b), where the respective beam diameters are $4\sigma_x = 3.47$ mm and $4\sigma_y = 3.41$ mm at a low pulse rate and $4\sigma_x = 3.45$ mm and $4\sigma_y = 3.37$ mm at a higher pulse rate. The optics are designed with phase profiles to emit a Gaussian beam of $4\sigma = 3$ mm in diameter.
With the optics in the cavity, the Gaussian beam diameters are $4\sigma_x = 3.09$ mm and $4\sigma_y = 2.92$ mm at a low pulse rate (Fig. 5(c)) and $4\sigma_x = 2.95$ mm and $4\sigma_y = 3.07$ mm at a higher pulse rate (Fig. 5(d)). Apart from the output beam sizes, which compare exceedingly well against the expected value, we also compare the optical brightness of the output laser beam for the two cavities. The brightness of a laser beam is defined as the power emitted per unit surface area per unit solid angle. This factor is particularly important as the brightness describes the potential of a laser beam to achieve high intensities while maintaining a large Rayleigh range for small focusing angles. For a source that has constant brightness, the source is said to be isotropic and the brightness is defined as\textsuperscript{11}:

$$B = P \left( \frac{\pi}{\lambda M^2} \right)^2.$$  \hspace{1cm} (7)

where $P$ is either the average or the peak power at the output, $\lambda$ is the wavelength and $M^2$ is the beam quality factor. Thus high brightness requires high output power while maintaining a sufficiently low beam quality. We experimentally determine the beam quality factor according to the ISO standard\textsuperscript{12,13} for each cavity and together with the measured data of the output power we determine the optical brightness of the system. We find that at a low pulse rate, the brightness of the system with the optics inserted is 2.34X larger than the empty cavity and for a higher pulse rate, the brightness of the system with the optics inserted is 3.50X larger than the empty cavity as is presented in Fig. 6.

---

**Figure 5:** The output profiles of the laser operating without the optics for a (a) low pulse rate and (b) a higher pulse rate. These are compared to the laser with the optics inserted for a (c) low pulse rate and (d) a higher pulse rate. The experimental Gaussian output sizes compare exceedingly well with the expected value.

**Figure 6:** The optical brightness of the laser system with the optics when compared to that of an empty cavity is 2.34X larger at a low pulse rate and 3.50X larger at a higher pulse rate.
5. CONCLUSION

We have successfully demonstrated a Gaussian output in a solid-state laser by the intra-cavity metamorphosis of a Gaussian beam at the output coupler to a flat-top beam on the opposing mirror. The idea is initially implemented external to the cavity by the use of two spatial light modulators where we transform some Gaussian beam into a flat-top beam and reverse propagate the flat-top back into a Gaussian distribution. We have shown that the initial Gaussian beam and the reverse propagated Gaussian match very well in size although noisy. Optical elements of the refractive variant are designed from the phase profiles addressed to the SLMs for intra-cavity use. We show an excellent agreement of the output Gaussian size with the expected value at a low and high pulse rate. We compare the brightness of the cavity with the optics against an empty cavity and we find an increase in optical brightness of 2.34X at a low pulse rate and 3.50X at a higher pulse rate.

ACKNOWLEDGEMENTS

We wish to thank Mrs Liesl Burger for the invaluable discussions and useful advice.

REFERENCES