Speckle evolution with multiple steps of least-squares phase removal

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We study numerically the evolution of speckle fields due to the annihilation of optical vortices after the least-squares phase has been removed. A process with multiple steps of least-squares phase removal is carried out to minimize both vortex density and scintillation index. Statistical results show that almost all the optical vortices can be removed from a speckle field, which finally decays into a quasiplane wave after such an iterative process.

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I. INTRODUCTION

In stochastic optical fields, especially optical speckle fields [1], there are many dark points with vanishing intensity and undefined phase, around which the phase changes by 2π along a closed loop and forms a local helical wave front. These points are so-called phase singularities or optical vortices [2–4]. Due to these optical vortices, a speckle field is often regarded as an optical vortex field. The statistical properties of a speckle field in the transverse two-dimensional plane have been well studied [5–9]. Although these vortices are associated with points of zero intensity, they are very stable topological structures that cannot be removed or destroyed by any perturbation in the local background fields. Considering the speckle field in a three-dimensional volume [10–13], we see that these point-like optical vortices form many infinitely long lines or closed loops, which can be tangled with each other [14]. In fact, a recent numerical study found that about 27% of the vortex lines in speckle fields with a Gaussian spectrum form closed loops [15]. Due to the conservation of topological charges, optical vortices can only be created and annihilated in pairs at points where the vortex lines curve backward or forward along the direction of propagation.

Such fully developed speckle fields are similar to the final field that is obtained after an optical beam propagates over a long distance through a severely turbulent atmosphere. The overall scale factor of the field structure, the coherence area A_c, is static. In spite of the complicated topology of these vortex lines in a fully developed speckle field, the rate of vortex creation along the direction of propagation is statistically equal to the rate of vortex annihilation. In this sense, the vortex density, which is defined as the total number of vortices per unit cross-section area and given by half the inverse of the coherence area of the speckle field, $D_v = 0.5A_c$ [6,8], remains unchanged throughout the whole volume of the speckle field. Moreover, the scintillation index, defined as $S_I = \langle I^2\rangle/\langle I\rangle^2 - 1$, where $\langle\cdot\rangle$ means the ensemble average, is always equal to 1 throughout the whole volume of the speckle field. On these grounds, a fully developed speckle field is in a state of equilibrium, although the field changes randomly in the local areas, i.e., the vortex lines have a large-scale self-similarity characteristic of a Brownian random walk [15], during its free-space propagation.

In our recent work [16], we studied the evolution of optical vortices and the scintillation index in a speckle field when its equilibrium state is destroyed by removing the continuous part of the phase. We found that the vortex density and the scintillation index of the phase-perturbed field change during its free-space propagation. Such a phase-perturbed speckle field eventually restores equilibrium with a lower vortex density after propagating far enough. Based on this work, we now study the evolution of a speckle field using multiple steps of continuous phase removal. Three quantities are used to monitor the evolution of a nonequilibrium speckle field. The first quantity is the vortex density D_v as described above. The second quantity is the scintillation index. A very small scintillation index indicates small fluctuations in the intensity with no vortices in the field, while a large scintillation index indicates numerous vortices in the field. The third quantity is the probability density function (PDF) of intensity $P_I(I)$, which is a negative exponential function $[1/\langle I\rangle \exp(-I/\langle I\rangle)]$ when the field is a fully developed speckle field [1,17] (i.e., in a state of equilibrium).

In this paper, we report, first, the observation of the dip in the vortex density, as well as in the scintillation index, as a function of propagation distance. This reveals a hitherto unknown behavior of stochastic light. We then present some procedures that we use in an attempt to understand this phenomenon and to remove optical vortices from the field. What we present is but one possible way to probe this complex behavior. A procedure for removing vortices from fields is critical in order to implement adaptive optics in free-space optical communication, where the vortices are induced on propagation through atmospheric turbulence. In principle, the method we describe below could achieve complete removal using cascade adaptive optics systems, although at present the technological implementation is both difficult and expensive. However, a greater understanding of the vortex elimination procedures using conventional wave-front sensing and adaptive optics may lead in the future to a practical free-space optical communication system with a very low bit error rate.

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The remainder of the paper is organized as follows. In Sec. II, we describe and review the evolution of vortex density and scintillation index after the continuous part of the phase has been removed from a speckle field. In Sec. III, we describe the method of multiple steps of least-squares phase removal. The least-squares phase removal is repeated for several times at some desired propagation distances where the vortex density of the field reaches its minimum value. In Sec. IV, numerical simulations are carried out and statistical results, such as vortex density and PDFs of intensity, are provided and discussed. Section V gives some discussions and conclusions.

II. REMOVING THE LEAST-SQUARES PHASE FROM A SPECKLE FIELD

A fully developed speckle field is obtained in the far field after scattering coherent light through a highly distorted medium or reflecting coherent light from a surface with its roughness on the scale of or greater than a wavelength. The typical intensity and phase of such a speckle field are shown in Figs. 1(a) and 1(b), respectively. Such a speckle field $\psi(\mathbf{x})$ can therefore be written as

$$\psi(\mathbf{x}) = |\psi(\mathbf{x})|e^{i\theta(\mathbf{x})} = \iint_{-\infty}^{\infty} \alpha(\mathbf{a})e^{-i2\pi\mathbf{a}\cdot\mathbf{x}}\, d^2a, \tag{1}$$

where $\mathbf{x}$ is the two-dimensional position vector on a plane perpendicular to the direction of propagation, which is assumed to be the z axis, $\mathbf{a}$ is the two-dimensional spatial frequency vector, and $\alpha(\mathbf{a})$ represents the random angular spectrum for the speckle field. In this paper a Gaussian spectral envelope is used to restrict the random angular spectrum to the area around the origin, $\alpha(\mathbf{a}) = \tilde{\chi}(\mathbf{a})\exp(-|\mathbf{a}|^2/w^2)$, where $\tilde{\chi}(\mathbf{a})$ is a normally distributed complex-valued random function and w is a scale for the radius of the random angular spectrum. The spatial coherence length L_c, which is here defined as the square root of the spatial coherence area A_c, is inversely proportional to w [8].

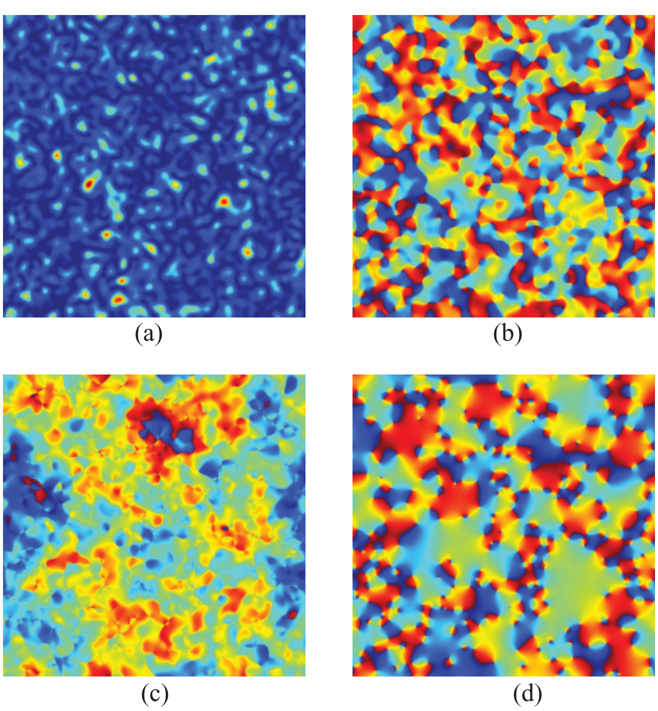

FIG. 1. (Color online) (a) Intensity, (b) total phase, (c) least-squares phase, and (d) singular phase of a speckle field.

The phase of a speckle field can be separated into two parts, as shown in Figs. 1(c) and 1(d): a continuous phase $\theta_c(\mathbf{x})$ and a singular phase $\theta_s(\mathbf{x})$,

$$\theta(\mathbf{x}) = \theta_c(\mathbf{x}) + \theta_s(\mathbf{x}), \tag{2}$$

where $\theta_s(\mathbf{x})$ is a sum of phase singularities, which can be expressed as

$$\theta_s(\mathbf{x}) = \arg\left\{\prod_{n=1}^{N} e^{i\nu_n\phi(\mathbf{x}-\mathbf{x}_n)}\right\}, \tag{3}$$

where $\arg\{\cdot\}$ denotes the phase, $\phi(\mathbf{x}-\mathbf{x}_n)$ represents a phase singularity located at $\mathbf{x}_n$, with the definition $\phi(\mathbf{x}) = \arctan(y,x)$, and ν_n represents the topological charge (±1) of the phase singularity. High-order vortices cannot exist in a fully developed speckle field because they are not stable and will decompose into elementary vortices with unit topological charge [18]. With such a separation, one can have

$$\begin{gathered} \nabla\times\nabla\theta(\mathbf{x}) = \nabla\times\nabla\theta_s(\mathbf{x}) = \pm2\pi\delta(\mathbf{x}-\mathbf{x}_n), \\ \nabla\times\nabla\theta_c(\mathbf{x}) = 0, \end{gathered} \tag{4}$$

where $\nabla\times$ denotes the curl, ∇ denotes the gradient, and $\delta(\cdot)$ is a two-dimensional Dirac delta function.

According to Eqs. (2) to (4), the continuous phase $\theta_c(\mathbf{x})$ is curl free and therefore does not contain any phase singularities. The separation between the continuous part and the singular part of the phase is, however, not unique. In principle one can calculate the continuous phase numerically by subtracting the singular phase from the initial phase by first locating all the phase singularities and then composing the singular phase function as in Eq. (3). However, since there are typically hundreds or even thousands of vortices in a speckle field, it is difficult to locate these phase singularities precisely in such an experimental setup [19,20]. Furthermore, the initial phase of the incident speckle beam is also unknown.

Usually, the wave front of an incident beam can be measured by a wave-front sensor. From the output of the wave-front sensor, which represents phase differences or phase slopes, one can estimate the incident wave front by using a least-squares algorithm [21,22]. In these wave-front estimation methods the singular phase was viewed as part of the measurement error or noise, which was neglected, until it was later pointed out by Fried [23]. It then follows that another way to separate the continuous phase $\theta_c(\mathbf{x})$ is to compute the least-squares phase by using a Fourier transform [24–26], as shown in Fig. 1(c). This gives an optimal estimate of the continuous part of the total phase $\theta(\mathbf{x})$,

$$\theta^{LS}(\mathbf{x}) = -\mathcal{F}^{-1}\left\{\frac{\mathcal{F}\left\{\nabla_T^2\theta(\mathbf{x})\right\}}{|\mathbf{a}|^2}\right\}, \tag{5}$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ represent the fast Fourier transform and its inverse, respectively, and ∇_T^2 represents the transverse Laplacian operation.

By multiplying a speckle field with the complex conjugate of its least-squares phase, one can remove the continuous part of the phase. This is similar to measuring the distorted wave front with a wave-front sensor, reconstructing the wave front by some least-squares reconstruction methods, and then removing the reconstructed wave front (continuous part of the phase) by a deformable mirror, as in an adaptive optics system. As a result the speckle field now only contains the singular part of the phase, as shown in Fig. 1(d), with its intensity unaffected. This can explain the reason why the traditional adaptive optics system cannot correct the strongly scintillated beams effectively. The vortices are indicated in Fig. 1(d) by the points where all the phase values (denoted by the different colors) come together. The resulting field is called the phase-perturbed speckle field, and the equilibrium in this field is destroyed.

However, the evolution of the vortex density of such a phase-perturbed speckle field is not a simple exponential decay as one may intuitively expect. Along the propagation direction, this phase-perturbed field experiences two stages of evolution. The first stage is a decay process. The rate of vortex annihilation is higher than the rate of vortex creation, which leads to a decrease in the number of vortices in the field. At the same time, one also finds that the scintillation index decreases as shown in Fig. 2(a). The PDF of intensity, which is shown in Fig. 2(b), changes from an initial negative exponential function at $z = 0$ into a semi-Gaussian function near $z = z_m$, where the vortex density reaches a minimum value.

The second stage is a self-regeneration process. The rate of vortex annihilation is lower than the rate of vortex creation, which leads to an increase in the number of optical vortices in the field. However, the initial rate of decrease in both vortex density and scintillation index is an order of magnitude higher than the subsequent rate of increase. The scintillation index finally increases back to 1 as $z \to \infty$, which means that the equilibrium state is restored and the field returns to a fully developed speckle field. The PDF curve for $z \to \infty$ has the same negative exponential function, as shown in Fig. 2(b), which also indicates that an equilibrium state is restored and established in the final field. However, the ability of this regeneration is limited because the final vortex density as $z \to \infty$ is about 12% lower than the initial value [16]. The final field can be viewed as a new fully developed speckle field with a different equilibrium state.

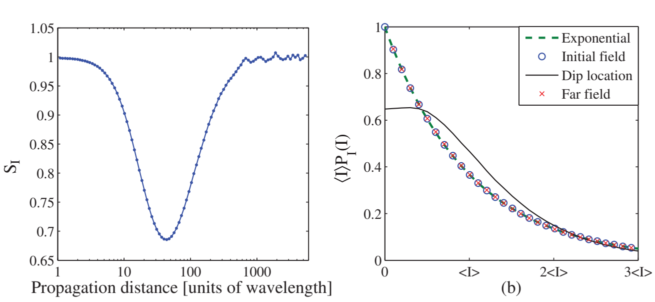

FIG. 2. (Color online) (a) Evolution of scintillation index after one step of least-squares phase removal and (b) PDFs of intensity at different propagation distances (dashed curve denotes the theoretical negative exponential distribution for the fully developed speckle field).

III. MULTIPLE STEPS OF LEAST-SQUARES PHASE REMOVAL

According to the discussion above, the evolution of the phase-perturbed speckle field shows a combination of a decay process and a self-regeneration process. This new speckle field can be characterized by its coherence area A_c, which can be calculated by

$$A_c = -\frac{2\pi}{W''_{\mathbf{x}=0}}, \tag{6}$$

where W'' is the magnitude of the transverse spatial second derivative of the autocorrelation function of the new speckle field [5].

The Wiener-Khinchin theorem implies that W'' can be computed from the Fourier transform of the power spectral density. In fact, the autocorrelation function of a deterministic beam in free space is independent of the propagation distance, from which it immediately follows that the initial phase-perturbed field and the final speckle field have the same autocorrelation function. Hence, one can predict the final vortex density D_v of a phase-perturbed speckle field with the knowledge of the autocorrelation function at $z = 0$, which is given by

$$D_v = -\frac{W''_{\mathbf{x}=0}}{4\pi}. \tag{7}$$

The vortex density reaches a minimum value that is lower than those in the initial and final fields. The propagation distance where the minimum vortex density is obtained can be predicted by [16]

$$z_m = 1.1\frac{A_c}{\lambda} = -\frac{2.2\pi}{\lambda W''_{\mathbf{x}=0}}, \tag{8}$$

where λ is the wavelength.

After the phase-perturbed field has propagated over a distance of z_m through free space, it contains fewer vortices but more phase fluctuations due to the annihilation of vortices. This suggests that one can further reduce the vortex density in the field by removing the least-squares phase $\theta^{LS}_{z=z_m}$ at $z = z_m$, as calculated with Eq. (5). Thus a new phase-perturbed field $\psi_{z=z_m}$ is produced, ready to be propagated through free space for a further reduction in the number of vortices. As expected, the vortex density of this new field $\psi_{z=z_m}$ also experiences a decrease and subsequent increase in the vortex density during its propagation. The propagation distance z_{m1} where the new minimum vortex density is obtained can also be calculated with Eq. (8).

Such a process of least-squares phase removal and free-space propagation can be repeated multiple times, as shown in Fig. 3. Through such an iterative process virtually all the optical vortices can be removed from the speckle field. Note that z_{mn} changes for every successive step because the autocorrelation function W changes as a result of the reduction in the number of vortices.

IV. NUMERICAL SIMULATIONS

Numerical simulations are carried out to demonstrate the evolution of a speckle field with such multiple steps of least-squares phase removal. In the simulations the initial speckle field $\psi_{z=0}$ is represented by an array consisting of

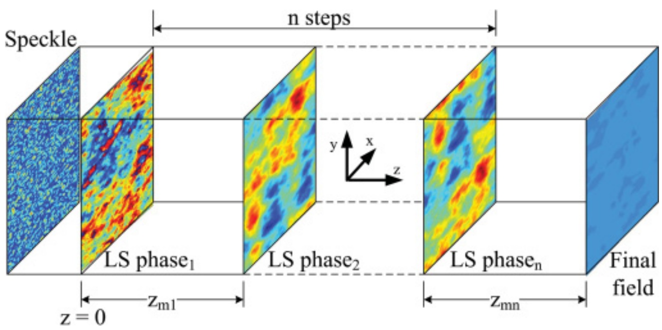

FIG. 3. (Color online) Multiple steps of least-squares phase removal.

512×512 samples. The Fourier transform of the random angular spectrum in Eq. (1) produces the initial speckle field with periodic boundary conditions. Similarly, the least-squares phase, calculated with Eq. (5), also satisfies periodic boundary conditions. Therefore, the resulting optical fields do not expand during propagation. The free-space propagation of the input speckle field is simulated with a numerical beam propagation method from Fourier optics [27]. At each propagation distance given by Eq. (8) the vortex density is determined by locating all the vortices inside the field. At the same time, the scintillation index and the PDFs of intensity are also calculated from the intensity of the field. The wavelength in the simulations is chosen small enough to ensure that the propagation stays within the paraxial limit. Hundreds of simulations are carried out to produce statistical curves of the vortex density, the scintillation index, and the PDFs of intensity.

The evolution of the vortex density is shown in Fig. 4(a) as a function of the number of phase correction steps. For this case the radius of the random angular spectrum is set to $w = 32$ with the units defined as one sample spacing in the Fourier domain. The averaged total number of vortices in the initial speckle fields is about 1600. The vortex density D_v is normalized by the initial number of vortices in each simulation. In Fig. 4(a) we see that the vortex density decreases rapidly as a function of the number of phase correction steps. After 100 such steps almost all of the vortices in the field are removed. We also note that the rate of decrease in vortex density becomes slower after about 10 steps of phase correction.

Figure 4(b) shows the same evolution of the vortex density as a function of the propagation distance. The required

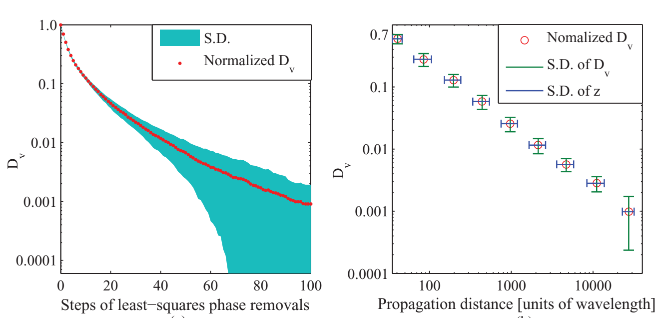

FIG. 4. (Color online) Vortex density vs (a) steps of least-squares phase removal and (b) propagation distance. Shaded cyan area and error bars denote the standard deviations (S.D.).

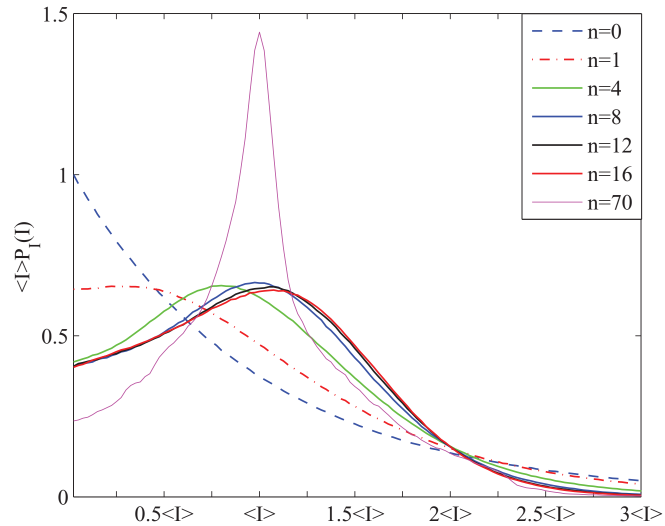

FIG. 5. (Color online) Probability density functions of intensity after different steps (n) of least-squares phase removal.

propagation distance between phase removals increases logarithmically with each step. The result is that the vortex density decreases according to a power law as a function of propagation distance. In some cases one or two vortex dipoles still remain in the final field. The separation distances between these remaining vortices are large compared to the size of the numerical array of the field. The theoretical annihilation distance required for such vortex dipoles is relatively large [28]. Therefore, it is challenging to remove such vortex dipoles.

In Fig. 5 we show the PDFs of intensity after different steps of correction. One can see that the initial PDF of the speckle field is given by a negative exponential function. After one step of correction, the PDF has a semi-Gaussian form, which is the same shape that is shown in Fig. 2(b) for the dip location at $z = z_m$. Since we repeat the least-squares phase removals, the PDF continuously transforms, with its value at $I = 0$ dropping down step after step.

The PDFs from one to ten steps of least-squares phase removal are close to modified Rician statistics with some differences introduced by numerical simulations. As pointed out by Goodman, the vortex density drops down exponentially with the ratio of the specular component to the random component in a field that has modified Rician statistics [1]. One may view the least-squares phase removals as a process in which it transfers the random components of the field into the specular components of the field. The observed evolution of the vortex density as shown in Fig. 4(a) seems to be consistent with this interpretation. However, whether the phase removals would result in modified Rician statistics still needs further investigation.

Those PDFs between 10 and 50 steps of least-squares phase removal become very similar, which indicates the slower drop down in the vortex density after 10 steps of phase removal. The speckle field turns into a vortex-free field after about 50 steps of phase removal. Since it is a vortex-free field, more steps of least-squares phase removal will further remove the fluctuations in both amplitude and phase and finally turn the field into a quasiplane-wave field. In Fig. 5 one can see that a peak is formed at $I = \langle I \rangle$ in the PDF curve when

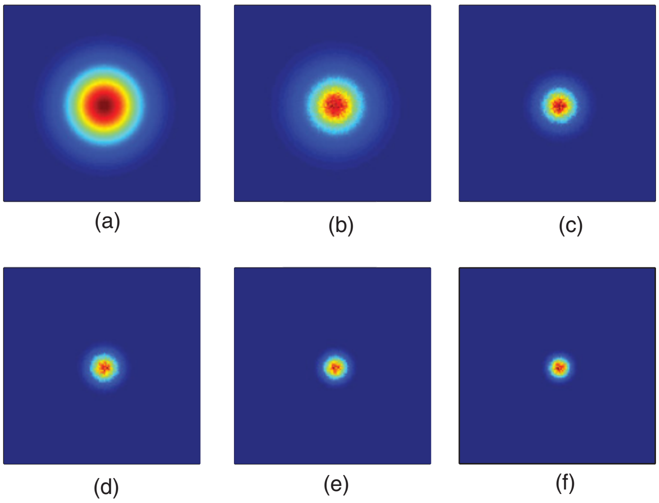

FIG. 6. (Color online) Point spread functions after (a) $n = 0$, (b) $n = 1$, (c) $n = 4$, (d) $n = 8$, (e) $n = 12$, and (f) $n = 16$ steps of least-squares phase removal.

70 steps of least-squares phase removal have been applied, which indicates that a quasiplane wave has been achieved.

The point spread function can reveal the degree of blurring of the point object, which is a measure for the quality of an imaging system. The evolution of the point spread functions after repeated steps of least-squares phase removal is consistent with the evolution of the PDFs. As shown in Fig. 6, the size of the central peak of the point spread function becomes smaller and smaller when more steps of phase removal have been applied. The point spread function of the initial speckle field ($n = 0$), as shown in Fig. 6(a), has the biggest size of the central peak. After about ten steps, the improvement in the central peak becomes less efficient. One can expect that the point spread function would approximate an Airy disk when the field becomes a quasiplane wave with all of the vortices having been removed through numerous steps of least-squares phase removal. However, with a traditional adaptive optics system, one cannot remove those vortices in the strongly distorted optical field because one step of least-squares phase removal with the deformable mirror only removes the continuous part of phase distortions, as illustrated in Fig 6(b).

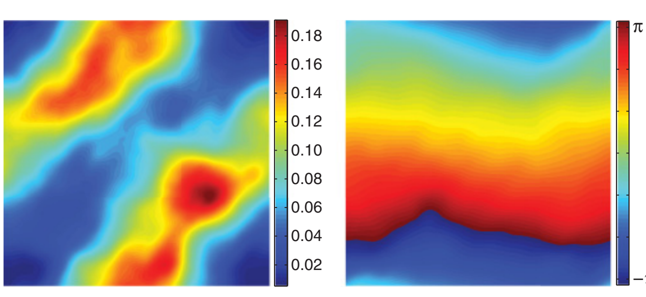

FIG. 7. (Color online) (a) Intensity (normalized by the maximum intensity of the initial field) and (b) phase (without vortices) of the final field after 53 steps of least-squares phase removal have been applied on a speckle field initially with 1658 vortices. See Supplemental Material [29] for the whole evolution procedure.

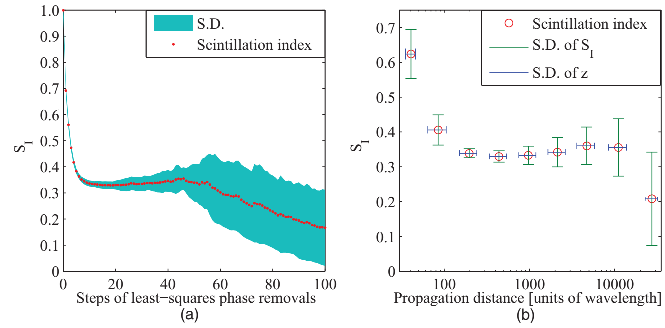

FIG. 8. (Color online) Scintillation index vs (a) steps of least-squares phase removal and (b) propagation distance. Shaded cyan area and error bars denote the standard deviations (S.D.).

Figure 7 shows the intensity and phase of a final field after 53 steps of least-squares phase removal, applied to a speckle field that initially had 1658 vortices. We find that all the vortices are removed from the speckle field. The intensity fluctuations of this vortex-free field are still large, as indicated by the scintillation index of 0.29.

In most cases the scintillation index that is obtained just after all of the vortices are removed from the speckle field is rather large because the intensity fluctuations in these vortex-free fields are coupled into both the amplitude and the phase during further propagation. Further reduction of the scintillation index of such a vortex-free field requires more least-squares phase removal steps. However, due to the absence of vortices, the required propagation distance z_m, given by Eq. (8), cannot be computed any longer. Here we keep z_m fixed at its last value where there were still some vortices left. The evolution of the scintillation index with this modification is shown by the curves in Fig. 8. The simulation results show that the decision to keep z_m fixed is a successful strategy because the scintillation index is further reduced during subsequent steps of least-squares phase removal.

We find that the averaged scintillation index decreases rapidly from 1 to 0.35 over the first ten steps, which corresponds to the rapid decrease in the vortex density from 1 to 0.1. The flatness or the slight increase in the scintillation index after the first ten steps until about the 50th step can be explained as follows. Initially, the scintillation index is strongly affected by the vortex density due to the zero intensity at the vortex cores. When the number of vortices decreases the fluctuations in the field also start to play a role. When there are no vortices left, the removal of the least-squares phase can reduce the fluctuations of the field as in an adaptive optics system. As a result, the scintillation index is further reduced, as shown by the curve in Fig. 8(a) beyond about 50 steps.

V. DISCUSSION AND CONCLUSIONS

For monochromatic light, the dark areas (optical vortices) that appear in the speckle fields extend throughout the whole volume of the field. They are very difficult to manipulate or remove. In this paper, we strived to elucidate a physical phenomenon associated with the way in which a phase-perturbed speckle field evolves. Removing the least-squares (continuous) phase of a speckle field, one finds that the optical vortices in the speckle field are reduced through pairwise annihilation at a rate that exceeds the pairwise creation of vortices. Moreover, the resulting curve of the vortex density follows a particular shape, which contains a dip at a characteristic distance scale that depends on the coherence area and the wavelength. We also showed that, by repeating the least-squares phase removals at subsequent dips, one can eventually remove virtually all the optical vortices from the optical field.

In addition to the dip in the curve of the vortex density, we also observe a dip in the curve of the scintillation index. The mechanisms for these dips are not understood at the moment. With further investigation, we hope to understand these phenomena to the extent that one can predict the behavior of the vortex density (and the scintillation index) in any stochastic optical field.

Note that these vortices are not removed directly, but indirectly through the removal of the continuous (least-squares) part of the phase function. With multiple steps of least-squares phase removal a speckle field with a large number of vortices can be transformed into a quasiplane wave without vortices. The fact that it is, in principle, possible to remove optical vortices from a speckle field in this way implies that there is a strong connection between the dynamics of the optical vortices and the continuous part of the phase function, albeit one that is still poorly understood. A better understanding of this connection could lead to better control of optical vortices in stochastic optical fields, which can, in turn, lead to a breakthrough in an approach to correct strongly scintillated optical fields. An understanding of the dynamics of optical vortices in stochastic optical fields can also aid the research in fields such as optical trapping and optical communication based on optical vortices (orbital angular momentum).

This procedure can, in principle, remove all the vortices from a speckle field. If such an adaptive optics system can be implemented physically, one can expect to achieve a remarkable improvement in performance over conventional adaptive optical systems that employ only one least-squares phase removal. Although this method is intuitively appealing, the large number of phase removals and the large propagation distances (which previously caused the authors to be unable to reduce the vortex density down to zero in strongly scintillated beams [30]) may present some challenges in a physical experiment. However, one can see that the study in this paper for a speckle field with thousands of vortices might be the worst case that needs to be corrected with an adaptive optics system. Actually, three or four steps of least-squares phase removal can remove half of the vortices from the speckle field, which already represents a significant improvement in the system performance.

ACKNOWLEDGMENT

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