Transverse correlation vanishing due to phase aberrations

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Abstract:
The concept of transverse correlation vanishing, between the fraction of power contained in the centre and wings of a probe beam, recently introduced [Opt. Commun. \textbf{282} (2009) 3854-3858] is important to be considered when measuring the effective focal length of any refractive index profile. When the latter is not parabolic it is shown that the transverse correlation could vanishe, and this represents a potential source of error when measuring any lensing effect. We propose a low cost set-up (two photodiodes, a pinhole and a stop) able to reveal if the probe beam has monitored a pure or an aberrated lensing effect especially when it is time-dependent.

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1. Introduction

Several kinds of induced optical inhomogeneities in the active material are detrimental to efficient laser operation. Among them, one can quote thermal lensing (TL) [1], Kerr-lens [2] and population lensing effects [3]. These lensing effects are generally measured by monitoring the change in the geometrical characteristics of a Gaussian probe beam deduced from the change in the transmission after a pinhole or an opaque disk (beam-stop) set in the far-field [3-6]. The beam-stop and pinhole techniques have different sensitivities, but they should lead to the same measured focal length. This can be easily understood if one notes that the fraction of power contained in the centre and the wings of a Gaussian beam are correlated. Here, (transverse) correlation means that power contents in the centre and the wings of the probe beam are complementary: if one is increasing due to a beam divergence variation the other is decreasing. It was recently demonstrated that the correlation between the fraction of power contained in the centre and the wings of the probe beam in the far-field could vanish when some clipping, even weak, occurs in the near-field [7]. It has then been proved experimentally that this effect can be responsible to important errors when monitoring a population lensing effect since one could be mistaken about its sign, i.e. converging or diverging [8]. We speculate that such transverse correlation vanishing can also occur with phase aberrations, and this paper deals with its demonstration. Indeed, when the refractive index profile under study is not a parabola, the Gaussian probe beam after passing through the sample is no longer Gaussian in shape. Thus, one can inquire about what is measured through a pinhole or a stop set in the far-field knowing that both of the two causes (amplitude and phase) responsible of transverse correlation vanishing represent a potential source of error when measuring any lensing effect.

2. Divergence diagnostic based on a stop or a pinhole

Thermal effects can be time-dependent or constant in time, and their monitoring can be done by considering the variation of angular divergence of a probe Gaussian beam resulting from its propagation through the component under study. From such measurements, it is thus possible to deduce the corresponding variation of the beam-waist size, and hence to deduce the focal length of the lens responsible of this variation. It is worth noting that in this purpose a time resolved beam divergence diagnostic is needed. Figure 1 shows two such diagnostics,
the first (second) one consists in monitoring the transmission $T_p(T_s)$ of the pinhole (stop) of radius $r_p$ ($r_s$) set at a distance $L$ from the beam-waist plane of the Gaussian probe beam. The angular divergence $\theta_p$ ($\theta_s$) deduced from pinhole (stop) diagnostic is given by [7]:

$$\theta_p = \frac{1}{L} \sqrt{-\frac{2r_p^2}{\ln(1-T_p)}} \quad \theta_s = \frac{1}{L} \sqrt{-\frac{2r_s^2}{\ln(T_s)}}.$$ (1)

Before to proceed, it is important to precise the sense of the term “transverse correlation” that is used in the following. The transverse correlation for a pure Gaussian beam means that an increase in transmission $T_p(T_s)$ results from a laser beam divergence decrease (increase), and vice-versa. In this case, the values of $\theta_p$ and $\theta_s$ should be equal. It has been recently demonstrated that fractions of power in the centre and the wings in the far-field of a Gaussian beam are no longer correlated when it undergoes some near-field clipping [7]. In this case, the values of $\theta_p$ and $\theta_s$ are different as shown in Fig. 2, plotting the variations of ratio $\theta_p/\theta_s$ as a function of the beam-waist size $W_0$ for different near-field aperture radii $\rho_0$. The numerical calculations in Fig. 2 have been made for the parameter values given in Ref. [7]. This effect of transverse correlation vanishing can greatly affect the monitoring of any variable lensing effect. Indeed, the divergence diagnostic based on the pinhole points out for instance that we are dealing with a convergent lensing effect while the same beam monitored by the divergence diagnostic based on a stop indicates that the lensing effect is divergent [7]. Both diagnostics give the same value for $\theta_p$ and $\theta_s$ for $\rho_0 > 2.4W_0$. Recently, this concept of transverse correlation vanishing due to a near-field aperturing has allowed clarifying the sign of the population lensing effect [8] in a $Cr^{3+}:Al_2O_3$ laser.

We are interested here in investigating if a similar effect of transverse correlation vanishing could occur when monitoring a lensing effect accompanied with a phase aberration. The latter means that the phase shift $\varphi(\rho)$ introduced by the TL effect under study does not varies radially following a quadratic law but is more complicated. In the following we will consider a phase shift due to TL effect in the form:

$$\varphi(\rho) = A_1\rho + A_2\rho^2 + A_3\rho^3.$$ (2)
where $\rho$ is the radial coordinate. Note that aberrated thermal lenses are for instance usually encountered in diode-pumped solid-state lasers. Indeed, it is reported that the lateral pumping of a laser crystal can lead to a thermal axicon, namely the crystal is characterised by an axicon-like phase profile transmittance \cite{9} \( \tau(\rho) = \exp[-iA\rho] \), or to a lensing effect including spherical aberration \cite{10}.

The electric field of the Gaussian beam incident on the aberrated thermal lens to be measured is characterised by its unit-amplitude distribution given by

\[
E_{\text{in}}(\rho, z) = \frac{W_0}{W(z)} \exp\left(-\frac{\rho^2}{W^2(z)}\right) \exp\left[-i \left( kz - \phi(z) + \frac{k\rho^2}{2R_c(z)} \right) \right],
\]

where $W$ and $R_c$ represent the Gaussian beam radius and its radius of curvature at plane $z$, respectively. These quantities, as well as the Gouy phase shift $\phi$, are $z$ dependent and given by

\[
W^2(z) = W_0^2 \left[1 + \left(\frac{z}{z_R}\right)^2\right],
\]

\[
R_c(z) = \left[1 + \left(\frac{z_R}{z}\right)^2\right],
\]

\[
\phi(z) = \arctan(\frac{z}{z_R}),
\]

where $z_R = \frac{\pi W_0^2}{\lambda}$ is the Rayleigh range.

The transmission $T_p(T_s)$ of the pinhole (stop) of radius $r_p(r_s)$ set in the far-field at distance $L$ is obtained from the following power ratio

\[
T = \frac{\int_a^b |E_d(r, L)|^2 rdr}{\int_0^\infty |E_d(r, L)|^2 rdr},
\]

where $r$ is the radial coordinate in the plane $z=L$ where the pinhole (stop) is set, $a = 0 (a = r_s)$ and $b = r_p (b = \infty)$. The incident Gaussian field diffracts upon the phase profile $\phi(\rho)$, and the
resulting diffracted field distribution $E_d(r, L)$ in plane $z=L$ is given by the Fresnel-Kirchhoff integral
\[
E_d(r, L) = \frac{2\pi}{kL} \int_0^\infty \exp[-i\varphi(\rho)]E_m(\rho, 0) \exp\left(\frac{ik\rho^2}{2L}\right) J_0\left(\frac{2\pi}{L} r\rho\right) d\rho,
\]  
where $J_0$ is the zero-order Bessel function, and $k = 2\pi/\lambda$ the wave number.

Now, the effect of each pure aberration $\varphi_j = A_j \rho^j$ ($j=1$ to 3) on the phenomena of transverse correlation vanishing has to be characterised. For doing so, we will plot the variation of ratio $\theta_p/\theta_s$ as a function of coefficients $A_j$. In order to make possible the comparison of the effects of each aberration on the ratio $\theta_p/\theta_s$, the following condition are imposed:

\[
\varphi_1(\rho_{\text{max}}) = \varphi_2(\rho_{\text{max}}) = \varphi_3(\rho_{\text{max}}) = \varphi_0.
\]  

It is assumed that the phase aberration is set in the beam-waist plane of radius $W_0 = 1.5mm$. Arbitrarily, the value of $\rho_{\text{max}}$ is fixed to twice the incident beam width, $\rho_{\text{max}} = 2W_0$, where the intensity is only 0.03\% of the on-axis value. In the following we will express the aberration $\varphi_0$ in number of equivalent wavelengths given by the ratio $\delta_0 = \varphi_0/(2\pi)$. The variations of aberration amplitude $\delta_j(\rho) = \varphi_j(\rho)/(2\pi)$ as a function of the radial coordinate $\rho$ for $\delta_0 = 2\lambda$ is shown in Fig. 3.

The stop and pinhole of the divergence diagnostic (Fig. 1) are set in the far-field of the incident at a distance $L=10$ m from the beam waist plane of the incident probe beam which contains the phase aberration $\varphi(\rho)$. The pinhole and stop radii are chosen so that their initial transmission (without any phase aberration, i.e. $\varphi(\rho) = 0$) is equal to that used in the experiment (22\%).

**a) $\varphi_1(\rho) = A_2 \rho^2$:**

First of all, we must verify that ratio $\theta_p/\theta_s$ for $\varphi(\rho) = A_2 \rho^2$ is equal to unity whatever the value of $A_2$. Indeed, this can be expected since this phase profile is quadratic and thus corresponds to a pure lensing effect. As a result, the Gaussian probe beam remains Gaussian after going across the phase profile $\varphi(\rho) = A_2 \rho^2$ and the transverse correlation should be
total, i.e. $\theta_p = \theta_s$. For $\delta_0 = 2\lambda$, Fig. 4 shows that ratio $\theta_p / \theta_s$ is equal to unity, as expected, when $A_1$ varies. Thus we can conclude that the transverse correlation remains total when the probe Gaussian beam diffracts upon a parabolic phase distribution $\varphi_1(\rho) = A_1 \rho^2$.

b) $\varphi_1(\rho) = A_1 \rho$:

In this case the aberration phase profile is axicon-like and the diffracted probe beam is no longer Gaussian since precisely a linear phase profile is able to reshape the intensity profile [11]. As a consequence, one can expect that the use of equations (1), established for a Gaussian beam, leads to an error which should increase with $A_1$. This is confirmed by plots of Fig. 5 showing the variations of $\theta_p, \theta_s$ and $\theta$ versus $A_1$. Note that $\theta$ represents the divergence of the diffracted field emerging from the aberration phase profile $\varphi_1(\rho)$ and determined from

$$\theta = \frac{W_c}{L},$$

(10)

where $W_c$ is the effective width of the diffracted beam. This effective width which is convenient for characterising a non-Gaussian beam is defined as the radius of a circle that receives the same amount of power as the circle with a radius equal to the Gaussian beam width (i.e. 86.5%). It is seen in Fig. 5 that the value of $\theta_p, \theta_s$ and $\theta$ are about the same while $A_1 \leq 1.8 \text{mm}^{-1}$. This limit corresponds to an aberration amplitude over the whole probe beam of about $\delta(2W_0) = 0.86 \lambda$. For $A_1 > 1.8 \text{mm}^{-1}$ the divergence is overestimated due to the reshaping of the probe beam. Indeed, it has been shown [11] that the key parameter of the probe beam reshaping is the ratio $\Delta = \rho_\pi/W_0$, where $\rho_\pi$ is the radial distance so that $\varphi(\rho \pi) = A_1 \rho_\pi = \pi$. For $\Delta < 0.8$, i.e. $A_1 > 2.6 \text{mm}^{-1}$, the diffracted beam pattern becomes more and more hollow as $A_1$ increases. Now let us consider the variations of ratio $\theta_p / \theta_s$ versus $A_1$ shown in Fig. 6, which displays a reduction in the transverse correlation for $A_1 > 1.8 \text{mm}^{-1}$.

As a consequence, the diagnostic shown in Fig. 1 is able through the ratio $\theta_p / \theta_s$ to indicate if we are dealing with an aberrated or a pure lensing effect.

c) $\varphi_3(\rho) = A_3 \rho^3$:
It is observed in Fig. 7 that the aberration \( \varphi_j(\rho) = A_j \rho^3 \) has a little influence on ratio \( \theta_p/\theta_s \) and thus on transverse correlation. There is still to check the influence of the \( j=3 \) phase aberration on divergences \( \theta_p, \theta_s \) and \( \theta \) as shown in Fig. 8. It is seen that \( \theta_p, \theta_s \) and \( \theta \) are very close and this means that it should be difficult to reveal, for \( \delta_0 \) small, the presence of time-dependent \( j=3 \) aberration with the divergence diagnostic shown in Fig. 1. This effect can be understood if one considers the variations of \( \varphi_j \) as a function of the radial coordinate \( \rho \) shown in Fig. 3. Obviously, one can admit that the phase aberration has some influence on the spatial diffraction pattern even more as the phase shift is high where the intensity of the incident beam is high, i.e. for \( \rho < W_0 \). However, one can note that the amplitude aberration reduces around \( \rho = W_0 = 1.5 \text{mm} \) when \( j \) increases. To observe a significative effect on ratio \( \theta_p/\theta_s \) one has to greatly increase the value of \( \delta_0 \).

3. Experimental set-up

The experimental set-up shown in Fig. 9 consists of two parts:

The first one is made up of a He-Ne laser (633 nm) delivering Gaussian beam expanded by a 3X telescope (\( f_1=50\text{mm}, \ f_2=150\text{mm} \) ) before illuminating a programmable phase-only spatial light modulator (SLM) (Holoeye, PLUTO-VIS) with 1920x1080 pixels of pitch 8µm and calibrated for a 2\( \pi \) phase shift at 633nm. The incident beam radius on the SLM was \( W=1.5 \text{mm} \) and its angle of incidence was kept as small as possible (<15°). A diaphragm (\( D_1 \) ) was set after the SLM in order to get rid of the undesired diffraction orders, separated thanks to a grating displayed on the SLM in addition to the phase pattern. The role of the SLM was to reshape the wavefront of the incident beam following one of the phase profile \( \varphi_j(\rho) = A_j \rho^j \) (\( j=1 \) and \( 2 \)) with adjustable \( A_j \). The experimental study was restricted to a pure lensing effect (\( j=2 \)) and to the axicon-like aberration (\( j=1 \)) because, as shown in the previous section, its influence on ratio \( \theta_p/\theta_s \) is high and easily observable.

The second one was the divergence diagnostic (pinhole and stop) set in the far-field region of the beam diffracted upon the SLM. The only remaining diffraction order (undergoing the phase shift \( \varphi_j(\rho) \)) was then passed through a 0.16X telescope (\( f_3=250\text{mm}, \ f_4=40\text{mm} \) ) and focused (\( f_5=500\text{mm} \) ) on a circular opaque disk (stop S) and a pinhole (\( D_2 \)). The initial transmissions of the stop and pinhole were set to 22%. The transmitted signals were
finally collected by two photodiodes (Thorlabs, Inc.). The divergence diagnostic allowed the simultaneous determination of $\theta_p$ and $\theta_s$ as a function of $A_j$.

The experimental results are shown in Figures 10 and 11 which display the variation of the ratio $\theta_p/\theta_s$ as a function of $A_j$ (i.e. by varying the phase pattern displayed on the SLM), respectively for $j=1$ and 2. It should be recognised from these experimental (Fig. 11) and theoretical (Fig. 4) results that a good agreement is obtained for $j=2$. However, for $j=1$, agreement between experimental (Fig. 10) and theoretical (Fig. 6) results seems to be only qualitative. For instance, it is seen that the ratio $\theta_p/\theta_s$ has a maximum value of about 1.6 in the experiment, while it is only 1.15 in the simulation. In order to explain this discrepancy, let us examine the influence of the clipping that should result from the diaphragm $D_1$ set after the SLM (Fig. 9). The resulting diffraction can be characterised by the truncation ratio $Y_D = \rho_{D_1}/W$, where $\rho_{D_1}$ is the radius of diaphragm $D_1$ and $W=1.5$ mm the width of the Gaussian beam incident on the SLM. The theoretical modelling introduced in Section 2 is modified by replacing the infinite limit by $\rho_{D_1}$ as if the clipping should be set in the SLM plane. This assumption is just for simplifying the numerical calculations. The results are shown in Fig. 12 which displays the variations of ratio $\theta_p/\theta_s$ as a function of $A_j$ for several values of truncation ratio $Y_D$. It is seen that the clipping, even weak, resulting from diaphragm $D_1$ has a great influence on ratio $\theta_p/\theta_s$ and allows to reach a value of about 1.5 for $Y_D = 1.6$.

In order to realize that this clipping is very weak we have to consider the corresponding diaphragm transmission $(1-\exp[-2Y_D^2])$ which is equal in this case to 99.4%. In addition, the influence of the diaphragm clipping on ratio $\theta_p/\theta_s$ opens the possibility of increasing the sensitivity of the divergence diagnostic, shown in Fig. 1, for detecting the presence of phase aberrations. Note that in our experiment it has been not possible to suppress the clipping effects due to the diaphragm $D_1$ because the light spots associated with the different SLM diffraction order are too close.

4. Discussion

As previously mentioned, the phenomena of transverse correlation vanishing can be met with any lensing effect, which can be time-dependent. It is then important to have an efficient technique allowing to detect if the probe beam is subject to some phase aberration or to a perfect lensing effect especially in the transient regime. Note that sometimes thermal effects
can be characterised by a fast dynamic [12-14] that makes its characterisation more delicate. When the temperature profile is purely quadratic, we are dealing with a pure lensing effect, i.e. without aberration, which can be monitored by measuring the change in the geometrical characteristics of a collimated Gaussian probe beam. It is worth noting that in this case the probe remains Gaussian after passing through the thermal lens and the transverse correlation is total. The thermal lens can be monitored by measuring the variations of the transmitted power through a diaphragm or an opaque disk (Fig. 1), or to resort to lateral shear interferometry [15]. In contrast, in some cases the temperature profile in the laser rod can be very different from a parabola [9,16], and this leads to a degradation of the laser probe beam wavefront, and hence an increase in its propagation factor $M^2$. The Gaussian beam probe after passing through the laser is no longer Gaussian, and one can wonder about what is measured by a divergence diagnostic based on the use of a pinhole or opaque disk. A possible solution is to characterise the wave front emerging from the laser rod by its Zernike coefficients which can be determined by using a 2D Shack-Hartmann analyser [17-19], or from the reconstruction of the distorted phase pattern from three irradiance profiles recorded in different planes [20]. The above two methods are not well adapted to transient lensing due to the slow response of CCD cameras. It is important to note that when the pump beam is not continuous, the temperature profile in the laser rod evolves in shape with time [12,21,22]. This means that a transient lensing effect is characterised by time-dependent aberrations and this complicates the characterisation of transient thermal effects. In this case, it is really advisable to use the divergence diagnostics shown in Fig. 1 because it is a low cost set-up (two photodiodes, a pinhole and a stop) for which it is possible to reveal if the probe beam has monitored a pure or an aberrated lensing effect just by observing the evolution of divergence ratio $\theta_p/\theta_s$. This technique can replace favourably the use of a CCD camera or a Shack-Hartmann analyser in the case where the temporal sequence is too short, not for measuring the aberration but just to attest of the presence of aberrations able to distort the lens measurement.

Another case of interest concerns the thermal effects resulting from the longitudinal pumping of a laser crystal by a Gaussian beam of width $W_p$. In this case the resulting temperature profile is approximately parabolic in the central part ($r < W_p$) and is logarithmic for the rest [19,23-25]. As a consequence, depending on whether the width $W$ of the Gaussian probe beam is greater or smaller than the pumped area, one can be dealing with a pure thermal lens or an aberrated thermal lens. This can be easily solved from the variations of ratio $\theta_p/\theta_s$ as a function of $W$ the size of the probe beam. Indeed, a good choice of $W$ corresponds to a
perfect transverse correlation, i.e. $\theta_p = \theta_s$ which ensures that we are measuring a pure thermal lens.

In Ref. [7] it was demonstrated how a clipping due to a hard aperture can lead to a transverse correlation vanishing that represent a source of errors when measuring any lensing effect [8]. In practice this effect can be avoided by taking care of the diameter of all optical components that are crossed by the probing beam. In this paper, we have considered another cause of transverse correlation vanishing which is due to diffraction of the probing beam upon a non-quadratic aberration. This cause of transverse correlation vanishing is more difficult or even impossible to avoid since it is inherent to the refractive index profile under study. In addition, it is worth noting that “amplitude clipping” and “phase clipping” lead to very different far-field intensity patterns, and consequently to distinct transverse correlation vanishing features. In order to be convinced from this we have to compare the far-field intensity distribution resulting from:

(i) the amplitude clipping of a Gaussian beam incident on a circular diaphragm [7]
(ii) the phase clipping of a Gaussian beam incident on a circular $\pi$-plate [26].

The diaphragm and $\pi$-plate near-fields are made-up of non-Gaussian intensity distributions which look like similar [27]. In contrast, the diffracted beam profiles are different in the far-field region where is set the diagnostic divergence (pinhole/stop). Indeed, the far-field intensity distribution is about Gaussian, in the case of “amplitude clipping” [7], while it can be strongly distorted (i.e. reshaped into hollow or uniform pattern) in the case of “phase clipping” [26]. These two particular examples illustrate that in essence amplitude and phase clippings behave differently, and deserve to be considered through the effect of transverse correlation vanishing which can be responsible of systematic errors when monitoring any lensing effect. The coupling between the amplitude and phase perturbations that affect the transverse correlation of the probing beam is also a point worth considering in the future.

5. Conclusion

After having introduced [Opt. Commun. 282 (2009 3854-3858] the concept of transverse correlation vanishing when a Gaussian beam suffers a clipping even weak, we have demonstrated theoretically and experimentally in this work that a same effect can occur due to phase aberrations. The knowledge of loss of transverse correlation is essential when characterising any lensing effect otherwise the measurement could be wrong. The situation is more complicated in the case where we are dealing with a transient lensing effect, since it is
not so easy to characterise time-dependent aberrations. We propose a simple and low cost set-up (two photodiodes, a pinhole and an opaque disk) able to reveal if the probe beam has monitored a pure or an aberrated lensing effect even if it is time-dependent.

In this paper we have stressed on cylindrical geometry for sake of simplicity. However, many high power laser systems are based on the use of an edge-pumped slabs or disks which are rather than characterised by a rectangular geometry. Indeed, the strength of the thermal lensing is different in x- and y- directions. In addition, the distortion due to aberrations and consequently the transverse correlation vanishing can be different in the x- and y- directions. The method of study the transverse correlation proposed in this paper can be extended to the rectangular geometry by replacing the circular pinhole and stop set in the far-field by a transparent and opaque slits, respectively.

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REFERENCES


FIGURES CAPTION

Figure 1: Setup for the monitoring of variations of laser beam divergence.

Figure 2: Variations of far-field divergence ratio $\theta_p / \theta_s$ as a function of the Gaussian beam radius $W_0$ clipped by a near-field circular aperture of radius $\rho_0$. The angular divergence $\theta_p$ ($\theta_s$) is deduced from pinhole (stop) diagnostic (Fig. 1) by assuming the beam under study as perfectly Gaussian.

Figure 3: Variations of the amplitude aberration as a function of the radial coordinate.

Figure 4: Variations of divergence ratio $\theta_p / \theta_s$ characterising the crossing of a probe Gaussian beam through a phase distribution $\varphi(\rho) = A_2 \rho^2$. This case corresponds to a pure lensing effect for which the transverse correlation is total.

Figure 5: Variations of divergences $\theta$, $\theta_p$ and $\theta_s$ that characterise the crossing of a probe Gaussian beam through a phase distribution $\varphi(\rho) = A_4 \rho$.

Figure 6: Variations of divergence ratio $\theta_p / \theta_s$ for axicon-like aberration $\varphi(\rho) = A_7 \rho$.

Figure 7: Variations of divergence ratio $\theta_p / \theta_s$ characterising the crossing of a probe Gaussian beam through a phase distribution $\varphi(\rho) = A_7 \rho^3$.

Figure 8: Variations of divergences $\theta$, $\theta_p$ and $\theta_s$ that characterise the crossing of a probe Gaussian beam through a phase distribution $\varphi(\rho) = A_7 \rho^3$.

Figure 9: Experimental set-up involving a phase-only spatial modulator and a divergence diagnostic based on a pinhole and a stop set in the far-field. The focal length of the lenses are: $f_1 = 50mm$, $f_2 = 150mm$, $f_3 = 250mm$, $f_4 = 40mm$, $f_5 = 500mm$, $f_6 = f_7 = 40mm$. 

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**Figure 10:** Experimental variations of divergence ratio $\theta_p / \theta_s$ for axicon-like aberration $\varphi(\rho) = A_0 \rho$.

**Figure 11:** Experimental variations of divergence ratio $\theta_p / \theta_s$ for $\varphi(\rho) = A_0 \rho^2$.

**Figure 12:** Variations of divergence ratio $\theta_p / \theta_s$ for axicon-like aberration $\varphi(\rho) = A_0 \rho$ including a near-field clipping characterised by the truncation factor $Y_D$. 