THE COEFFICIENT OF WORK HARDENING IN STAGE IV

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Introduction

A processes of work hardening known as Stage IV is now becoming widely recognized [1,2]. While the conditions under which it is observed depend on temperature and on strain rate, the process itself appears to be completely athermal. The temperature independence of the rate of work hardening is strikingly illustrated by Figure 7(b) of Anongba et al. [1]. If Figures 2(b) and 6(b) of [1] are plotted on the equal scales (Figure 1), it is apparent that the regions corresponding to Stage IV of various temperatures join up to form a smooth curve.

![Graph showing work hardening rate normalized by modulus θ/μ against normalized flow stress τ/μ](image)

Figure 1 The graphs of work hardening rate normalized by modulus θ/μ against normalized flow stress τ/μ (Figures 2(b) and 6(b) of Anongba et al. [1], plotted on the same scale.)
Moreover, this curve is close to a parabola. The relation

\[ 10^4 \theta/\mu = 5.65 \left( 10^4 \tau/\mu - 0.95 \right)^{\frac{1}{3}} \] (1)

fits the interpolated curve within the errors of the interpolation.

This parabolic relation is also illustrated in Figure 4 of [2], and, for silicon and less convincingly, in Figure 2 of [2].

**The Mechanism of Work Hardening**

Argon and Haasen [2] have given a theory of this parabolic hardening process. Hardening results from the long-range stresses required to maintain the continuity of cells which undergo random relative rotations as plastic deformation proceeds. Each cell wall absorbs roughly equal numbers of dislocations of opposite sign coming from the cells on either side of it. Suppose a total number \( N \) of dislocations enters a cell wall. The shear strain \( \gamma \) is proportional to \( N \). Statistically, one expects to find an excess of about \( N^\frac{1}{3} \) dislocations of one sign, producing a random misorientation \( \psi \) across this wall which is proportional to \( N^\frac{1}{3} \). We thus expect to find (equation (21) of [2]),

\[ \psi = B \gamma^{\frac{1}{3}}. \] (2)

Argon and Haasen [2] estimated \( B \) from the observations of Rollett [3] on aluminium, and found

\[ B = 1.75 \times 10^{-2}. \] (3)

However, they noted that their calculated value of \( \theta/\mu = 1.05 \times 10^{-3} \) for copper was "somewhat high". Our estimate of \( \theta/\mu \) which follows is based on the microstructures observed at 727 K and 1145 K, where the value of \( \theta/\mu \) estimated from Figure 6(b) of [1] is about \( 7.5 \times 10^{-4} \). Since, according to the theory, \( \theta/\mu \) is proportional to \( B \), the experiments on copper are better fitted by

\[ B = 1.75 \times 10^{-2} \times \left( 7.5 \times 10^{-4} / 1.05 \times 10^{-3} \right) = 1.25 \times 10^{-2}. \] (4)

We suggest that the value of \( B \) is determined by the size of the cells, and that the value (4) is compatible with the cell size shown in the micrographs published by Anongba et al. [4].

Suppose \( N \) dislocation loops of Burgers vector \( b \) expand across a cell of linear dimensions \( L \). Then the plastic strain \( \gamma \) is

\[ \gamma = Nb/L. \] (5)

In a cross section in which each cell appears as a square of side \( L \), each loop adds one dislocation to two of the four walls of the cell, and so each wall has received a total of \( N \) dislocations, \( \frac{1}{2}N \) from each side. We thus expect a statistical residue of \( N^\frac{1}{2} \) dislocations per wall, leading to a misorientation

\[ \psi = N^\frac{1}{2} b/L. \] (6)
Thus we expect

$$B = \psi/\gamma = (b/L)^{\frac{3}{2}}. \quad (7)$$

Figures 7 and 15 of Anongba et al. [4] suggest that $L \approx 4.5 \times 10^{-6}$ m, while $b \approx 2.56 \times 10^{-10}$ m. Thus equation (6) predicts that

$$B \approx 7.5 \times 10^{-3}, \quad (8)$$

of the same order as the value (4) which reconciles the observations of [1] with the theory of [2].

Zehetbauer and Seumer [5] have shown that in Stage IV, $L$ is a very slowly decreasing function of strain. Then, from equation (7), $B$ will be a very slowly increasing function of stress, and, from equation (36) of reference [2], $\theta/\mu$ will increase with increasing $\gamma/\mu$ with a power slightly greater than $\frac{3}{2}$. In fact the curve drawn by eye through the Stage IV regions of Figure 1 passes at intermediate stresses somewhat below the parabola corresponding to equation (1), which is fitted at low and high stresses. Since the factor $g$ in equation (36) of reference [2] also increases with increasing stress, it is perhaps surprising that this deviation is not larger.

**Summary**

The theory of work hardening in Stage IV given by Argon and Haasen [2] depends on the relation (2) between the relative misorientation $\psi$ of neighbouring subgrains and the plastic strain $\gamma$. We suggest that $B$ is related to the subgrain size $L$ by equation (7), and show that this estimate is correct in order of magnitude.

**References**