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**Temporal terrain shading**

by

Antony K Cooper

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# TEMPORAL TERRAIN SHADING

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## ABSTRACT

This paper describes a suite of programs that simulates the lighting effects of the sun or moon on the earth's surface. A digital terrain model (DTM) is typically a two-dimensional regular tessellation of some measured phenomenon, the most common being elevation — such DTMs are known as digital elevation models (DEMs). The tessellation represents an area fixed to the surface of some celestial body, normally the earth. The suite of programs described here simulates the lighting effects of the sun or the moon by taking as input a DEM, a date and a time. The suite determines the position of the sun or the moon relative to the DEM and produces as output a DTM shaded accordingly. The treatment of shadows is also discussed. The resulting DTM may be used to paint a perspective view of the original DEM to provide a more realistic representation of the area.

## KEY WORDS

Digital Terrain Model, Digital Elevation Model, Sun Shading,  
Moon Shading

## INTRODUCTION

Temporally shaded terrain is an area that has been shaded according to the illumination it would receive from a particular source (the illuminant) at a particular moment. Typically, the terrain occurs on the earth's surface, and the illuminant is either the sun or the moon. The temporal information required to do temporal terrain shading consists of a date and a time.

## DIGITAL TERRAIN MODELS

Adapting from Edson & Denegre [1980] and Cooper [1985], we obtain the following definition:

A digital terrain model (DTM) is a model of some measured phenomenon occurring in an area fixed to the surface of a celestial body, which consists of discrete measurements of the phenomenon and a defined method to interpolate arbitrary values for the phenomenon in between the stored values. The most commonly measured phenomenon is elevation – such DTMs are known as digital elevation models (DEMs). Other measured phenomena include gravity, magnetic fields, temperature and pressure. DTMs normally occur on the earth's surface. The DTMs we use are two-dimensional regular tessellations, in particular, rectangular grids. Rectangular gridded digital data are also known as raster data. Other primary models of digital geo-referenced information, such as satellite imagery, are often in the form of raster data and may in fact be considered to be DTMs. A single stored value in raster data is also known as a pixel.

At the Centre for Advanced Computing and Decision Support, a few DEMs, supplied by the Chief Directorate: Surveys and Mapping, are available for research purposes. Their resolution per pixel varies between 80 metres and 500 metres.

## POSITION OF THE SUN AND MOON

### **Determining the unit vector of the sun's rays**

From Yallop [1978] we obtain the following formulae for the declination  $\delta_g$  of the sun (in degrees, to an accuracy of one minute), that is, we obtain the line of latitude above which the sun lies:

$$\delta_s = \arctan((0.43382 - 0.00027T) \sin(L - E)), \quad (1)$$

*where*

$$\begin{aligned}
E = & -\sin(L)(0.388 + 0.0593T - 0.00006T^2) \\
& -\cos(L)(1.802 - 0.0155T - 0.00086T^2) \\
& +\sin(2L)(2.487 - 0.0034T - 0.00004T^2) \\
& -\cos(2L)(0.006 + 0.0012T) \\
& +\sin(3L)(0.016 + 0.0025T) \\
& +\cos(3L)(0.081 - 0.0009T - 0.00004T^2) \\
& -\sin(4L)(0.053 - 0.0001T);
\end{aligned} \tag{2}$$

$$L = 279.697 + 36000.769T; \tag{3}$$

$$T = D/36525; \tag{4}$$

$$D = (\lfloor 365.25Y \rfloor + Z(J) + K + H/24); \tag{5}$$

$$\lfloor n \rfloor = \text{the integer part of } n; \tag{6}$$

$Z(n)$  = a function of the month as defined in the following table: (7)

Jan = -0.5	Feb = 30.5	Mar = 58.5	Apr = 89.5
May = 119.5	Jun = 150.5	Jul = 180.5	Aug = 211.5
Sep = 242.5	Oct = 272.5	Nov = 303.5	Dec = 333.5

(in a leap year, reduce the value for Jan and Feb by 1);

$$Y = \text{year in question} - 1900; \tag{8}$$

$$J = \text{month in question}; \tag{9}$$

$$K = \text{day of the month}; \tag{10}$$

$$H = \text{time in hours GMT}. \tag{11}$$

Using the declination of the sun  $\delta_s$ , the desired time and our position on the earth's surface, we can determine the angle of elevation to the sun  $h_s$  and the azimuth of the sun  $A_s$ :

$$h_s = \pi/2 - \theta \quad (12)$$

$$A_s = \arccos\left(\frac{\sin \delta - \cos \theta \sin \phi}{\sin \theta \cos \phi}\right), \quad (13)$$

where

$$\theta = \arccos(\sin \delta \sin \phi + \cos \delta \cos \phi \cos(15H + \lambda - 0.8)); \quad (14)$$

$$\phi = \text{line of latitude of observer}; \quad (15)$$

$$\lambda = \text{line of longitude of observer}; \quad (16)$$

Finally, the unit vector of the sun's ray  $\vec{S}$  is

$$\vec{S} = (\cos h_s \sin A_s, \cos h_s \cos A_s, \sin h_s). \quad (17)$$

### Determining the unit vector of the moon's rays

From Yallop [1978] we obtain the following formulae for the declination  $\delta_m$  of the moon (that is, we obtain the line of latitude above which the moon lies) and the Greenwich hour angle  $GHA_m$  of the moon (that is, we obtain the line of longitude above which the moon lies), both in degrees, to an accuracy of 1/10 of a degree:

$$\delta_m = \arcsin(\sin \varepsilon \cos \beta \sin \rho + \cos \varepsilon \sin \beta) \quad (18)$$

$$GHA_m = 99.691 + 36000.7689T + 15H - \arctan\left(\frac{\cos \varepsilon \cos \beta \sin \rho - \sin \varepsilon \sin \beta}{\cos \beta \cos \rho}\right), \quad (19)$$

where

$$\varepsilon = 23.452 - 0.00013Y; \quad (20)$$

$$\begin{aligned} \rho = & L + 6.289 \sin M - 1.274 \sin(M - 2N) \\ & + 0.658 \sin 2N + 0.214 \sin 2M \\ & - 0.186 \sin M' - 0.114 \sin 2F \\ & - 0.059 \sin(2M - 2N) - 0.057 \sin(M' + M - 2N) \quad (21) \\ & + 0.053 \sin(M + 2N) - 0.046 \sin(M' - 2N) \\ & + 0.041 \sin(M - M') - 0.035 \sin N \\ & - 0.03 \sin(M + M'); \end{aligned}$$

$$\begin{aligned} \beta = & 5.128 \sin F + 0.281 \sin(M + F) \\ & + 0.278 \sin(M - F) + 0.173 \sin(2N - F) \quad (22) \\ & - 0.055 \sin(M - 2N - F) - 0.046 \sin(M - 2N + F) \\ & + 0.033 \sin(2N + F); \end{aligned}$$

$$M' = 358.48 + 0.9856D; \quad (23)$$

$$L = 270.43 + 13.176397D; \quad (24)$$

$$M = 296.1 + 13.064992D; \quad (25)$$

$$N = 350.74 + 12.190749D; \quad (26)$$

$$F = 11.25 + 13.229350D; \quad (27)$$

with  $T$ ,  $H$ ,  $Y$  and  $D$  determined by equations 4, 11, 8 and 5 respectively.

Using the declination  $\delta_m$  of the moon, the Greenwich hour angle  $GHA_m$  of the moon and our position on the earth's surface, we can determine the angle of elevation to the moon  $h_m$  and the azimuth of the moon  $A_m$ :

$$h_m = \arcsin(\sin \phi \sin \delta_m + \cos \phi \cos \delta_m \cos GHA_m) \quad (28)$$

$$A_m = \arccos\left(\frac{\sin \delta_m \cos \phi - \cos \delta_m \cos GHA_m \sin \phi}{\cos h_m}\right), \quad (29)$$

with  $\phi$  given by equation 15.

Finally, the unit vector of the moon's ray  $\vec{M}$  is

$$\vec{M} = (\cos h_m \sin A_m, \cos h_m \cos A_m, \sin h_m). \quad (30)$$

## Using the illuminants position

After implementing these formulae, the results were checked against the *Astronomical Almanac* [1984] and the *Nautical Almanac* [1986] and were found to be sufficiently accurate.

For a DTM, the unit vector of the illuminant (the sun or the moon) is calculated for the centre pixel, and this unit vector is used for all the pixels in the DTM. The size of the DTMs (less than half a degree in either direction) does not warrant the time-consuming calculation of the unit vector for each pixel. For points up to a degree apart, the azimuth of the sun varies by less than six degrees and the elevation of the sun by less than two degrees.

Not all colours are equally visible — some have tones that are more easily discerned than others. The system used (an Ikonas RD3000 raster graphics system) has a palette of 256 colours, chosen from  $1024^3$  possible colours [Smit 1986]. Murch [1984] recommends the use of colour complements to distinguish between different areas. Two models are used: in the first we use blue for the shaded areas and yellow for the illuminated areas, and in the second model we use green and red respectively.

The number of tones allocated to the illuminated areas depends on the elevation of the illuminant, which approximately determines the ratio of illuminated to shaded pixels. For the shaded areas ambient light is simulated by using the slope at a pixel to select a tone — the steeper the slope, the darker the tone. The intensity of the tones for the shaded areas is obviously less than that for the illuminated areas. We have found this to give a good approximation for these areas.

## REFLECTION OF ILLUMINATION

There are a number of algorithms for determining the manner in which a surface reflects incident illumination. We use Lambert's cosine rule, which does not give specular highlights, as we feel that it is more appropriate to treat the earth's surface as a matte surface.

A matte surface reflects light with the same intensity in all directions. Lambert's cosine rule states that the reflected intensity is proportional to the cosine of the angle between the ray of incident light and the normal to the surface.

The slope at a pixel  $P_{x,y}$  is determined by the plane  $S$  joining the four nearest connected neighbour pixels, namely  $P_{x-1,y}$ ,  $P_{x+1,y}$ ,  $P_{x,y-1}$  and  $P_{x,y+1}$ . The surface normal at  $P_{x,y}$  is then the normal to  $S$ .

From Figure 1, one obtains

$$R = Ik \cos \theta, \tag{31}$$

where

$$R = \text{intensity of reflected light}; \tag{32}$$

$$I = \text{intensity of incident light}; \tag{33}$$

$$N = \text{surface normal at pixel}; \tag{34}$$

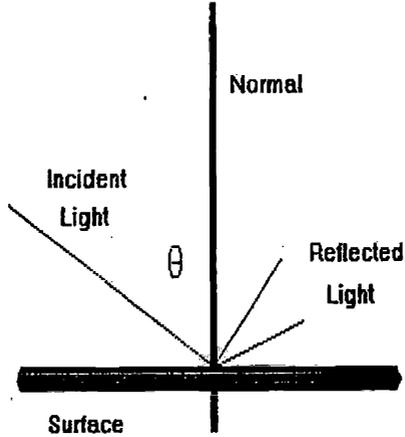


Figure 1: Illustration of Lambert's cosine rule

$$k = \text{constant describing reflectivity of surface;} \quad (35)$$

$$\text{with } 0 < k \leq 1;$$

$$\theta = \text{angle between incident ray and normal.} \quad (36)$$

If we normalize the incident light vector and the normal to the surface, we can use the dot product. Thus,

$$R = k(\vec{I} \cdot \vec{N}) \quad (37)$$

$$= k(I_x N_x + I_y N_y + I_z N_z).$$

For the purposes of temporal terrain shading, we set  $k = 1$ . This gives us the greatest possible range of shades to produce better pictures. When texturing (which requires detailed knowledge of the land cover) is incorporated,  $k$  must be defined accordingly.

Temporally shaded DTMs are generated for either the sun's illumination or the moon's illumination — if both are visible, the moon's contribution is so insignificant that it is not worth considering.

## PAINING PERSPECTIVE VIEWS

A temporally shaded DTM is not an end product in itself. Rather, it may be used to enhance other products, for example, perspective views of the original DEM. A number of algorithms for generating perspective views (see, for example, Robertson [1987]) are available. The temporally shaded DTM may be used to paint a perspective view in two ways:

- As the perspective view is generated, each pixel is repositioned and placed in the output display. Instead of writing the DEM's pixel value to that position, the temporally shaded DTM's value is written to that position in the output display.
- As the perspective view is generated, each pixel is repositioned and placed in the output display. Instead of writing the DEM's pixel value to that position, the *original coordinates* for the pixel are written to that position in the output display. Thereafter, *any* DTM may be used to paint the perspective view.

The second method is obviously preferable as it provides more flexibility — given a perspective view, one may paint it with any number of DTMs in quick succession. These DTMs could be used for time series analysis.

## ENHANCEMENTS

### Resolution of the DTMs

Temporal terrain shading attempts to represent the shading, by the sun or the moon, of a terrain as realistically as possible. Consideration must be given to the time taken by any method of enhancing the final product, as well as the effectiveness of the enhancement, given the limitations of the data.

At the Centre for Advanced Computing and Decision Support, the highest resolution for DTMs is 80 metres per pixel, which is adequate for our current suite of programs. In field tests, temporally shaded DTMs of the Cape Town area (consisting of 177 pixels East-West and 212 pixels North-South and bounded by 33°53' 49" South, 34°3'2"

South, 18°21'4" East and 18°30'12" East) were easily recognizable, and there was good correlation between the generated DTMs and the real world concerning those areas in light and in shade. However, while the correspondence was very good from a distance, from close up much detail was missing. This is due to the resolution of the data — any feature less than 40 metres wide would almost certainly be missed. Unfortunately, the costs involved in generating DTMs of fine resolution and covering a reasonably large area cannot be borne by research alone.

### **Land cover**

In the above-mentioned field tests, a few features were added to some of the DTMs, in particular, major roads, railway lines, rivers and dams. The linear features did not aid in correlating the DTMs to the real world (they tended to be hidden — much of the area is built up), but dams were easily identifiable (they are normally in the open and they are good reflectors).

These tests seem to indicate that temporally shaded DTMs would be enhanced by the use of land cover (especially areas) to provide colouring to the DTMs. The land cover could be used to moderate the intensity of the modelled reflected light by providing the  $k$  in equation 35. Those types of land cover that are good reflectors could be modelled using those algorithms that provide specular highlights (see, for example, Phong [1975]).

### **Shadows and reflections**

The suite of programs described here does not cater for shadows cast by the terrain, nor for reflections off illuminated areas into areas in shadow.

For each pixel, the surface at that pixel is either facing towards or away from the illuminant. Should it be facing away from the illuminant, it does not receive any direct illumination. However, even though it may be facing towards the illuminant, a pixel does not necessarily receive any direct illumination — it could be in the shadow of some other part of the DEM.

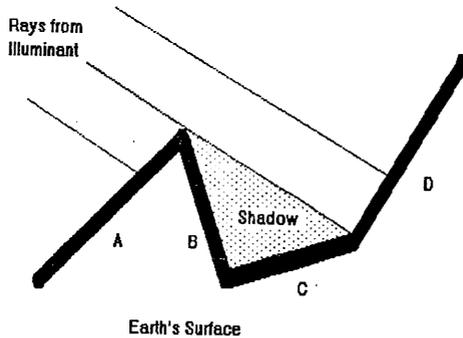


Figure 2: Illustration of shadows and shaded areas

In figure 2, *A* is receiving direct illumination, *B* faces away from the illuminant and receives no direct illumination, *C* faces the illuminant, but is in the shadow cast by *A* and thus receives no direct illumination. In our suite of programs, *C* would be coloured as though it had received direct illumination.

Shadows may be modelled by processing the DTM in the direction of the illuminant's rays while maintaining a mask of those pixels receiving direct illumination. Any pixels behind the mask are thus in the shadow of the mask and do not receive direct illumination. This process is similar to some of the methods used to generate perspective views. We have not implemented shadows as we do not feel it is justified given the coarse resolution of our DTMs.

In figure 2, light could reflect off *D* (which is directly illuminated) onto *B* and *C*. This indirect illumination could be modelled using the technique known as ray tracing or ray casting (see, for example, Whitted [1980]). Unfortunately, this method is very expensive computationally and not worth-while unless one has DTMs with a very fine resolution.

## CONCLUSIONS

This paper describes a suite of programs that simulates the lighting effects of the sun or moon on the earth's surface. The suite determines the orientation of the input DTM with respect to the illuminant and then, for each pixel, the intensity of the light reflected. The key factor in determining the success of the temporally shaded DTM that is produced is the resolution of the input DTM. Under moonlight, when one's vision is less perceptive, a resolution of 80 metres is probably adequate. However, under sunlight, one's vision is acute and a significantly finer resolution is needed.

Enhancements to the basic temporally shaded product are discussed and possible solutions for these enhancements are presented.

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