# Graph based Hydraulic Modelling of Pressure in Water Distribution Networks

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***Abstract*—The increased demands of water and high water losses in the past decade have brought a special attention to Water Distribution Networks. An efficient water distribution utilities management requires an adequate hydraulic modelling of the water network. Pressure analysis and control are key to the water management as they ensure a good delivery of water in the entire network by preventing water loss due to leakages. Traditional approaches of pressure control aiming to maintain a constant pressure in the entire network are deficient due to the continuous changes in water demands and therefore more effective methods of pressure control are needed. This paper proposes a hydraulic modelling of pressure for water distribution network. The pressure at the nodes in the network is modelled as graph equations to ensure a less complex pressure control and implementation. The results of this modelling show that to maintain an efficient water distribution, a pressure control mechanism is highly required. The simulations and discussions are presented for a case study were various control methods may be applied to monitor the pressure.**

***Keywords—Water distribution network; Hydraulic modelling; Graph equations; Pressure Control;***

## I. INTRODUCTION

The need for an efficient and effective pressure control mechanism in water distribution networks has been expressed recently [1], [2] and considering that water is a scarce resource, its efficient use and management is vital [3]. Controlling pressure is very important for water utilities as it is linked with the reliability and sustainability of the water infrastructures [4], [5]. This problem has been addressed since a long time [6][7][8][9][10] but is still a very important issue. Maintaining high pressure for a long period in an area of the network can damage the pipes and provoke leakages when the demands in water is relatively low [11], [12]. The volume of water that is supplied to the end users is considerably reduced compared to the volume of water entering the water network from the supply sources partially due to the lack of efficient pressure control in the network. The difference between those two volumes of water is referred by non-revenue water (NRW) [13], and is an issue that needs to be addressed by water management organisations.

In the traditional pressurised distribution network, the pump is designed and set to maintain a constant high pressure in order to supply water to the people at the far end of the pipe during peak hours [14]. Unfortunately, the high pressure reduces the lifespan of pipes, consumes more electricity during non-peak hours and if the constant pressure is low the delivery of water is not effective to some consumers; As a result, there is a need to have a pressure control mechanism. Pressure and flow sensors are used to monitor and control water flow rate in water distribution networks (WDNs) [15] and unfortunately the installation and maintenance cost of those sensors, is very high [16].

In order to control the pressure efficiently a hydraulic modelling is required. The hydraulic modelling is very important to understand and manage a wide and complex water network, and helps as well to determine the methods that need to be applied to control the pressure and the flow rate within a water network. A modelling of a hydraulic system should include all the network elements such as pipes, pumps, reservoirs, pressure reduction valves and the consumers' parameters [17]. Hydraulic modelling requires a high level detail of the network components, an easy links to all data sources and data output that can assist in the pressure control process, registration of records for future analysis and improvement, and powerful simulation engines for extension of simulation time [18]. The analysis of a water distribution network based on the number of the nodes is associated with two hydraulic variables, water consumptions (refers flow rates of pipes) and heads [19]. Therefore hydraulic analyses are performed by applying the fundamental laws of conservation of mass and conservation of heads.

Hydraulic pressure modelling in water distribution network is achieved by dividing the network into section of district meter area (DMA) to create discrete zone and avoid a huge amount of loss. Flow and pressure can be measured at different position in the DMA [20] and compute the water balance to determine the water loss based on the measurements. The subdivision of a network into DMA can be done using graph theory tools [21], multi-agent system [22] or complex network theory [23]. Graph theory techniques facilitate the representation and analysis of a large network. This approach ensures control of the network by providing a useful information on the connectivity of different nodes.

The aim of this paper is to present a graph theory based modelling of pressure in each district meter area for a large water network. This paper applies both the continuity and energy conservation principle in order to determine the pressure control decision. The remainder of this paper provides the related research work of the pressure modelling of a hydraulic system in section II and Sections III explains the proposed modelling and equations. Section IV presents the MATLAB simulation results and discusses the result to ensure pressure control for practical implementation while Section V concludes the paper and gives the recommendation for future works.

## II. Related Work

Hydraulic networks are modelled either in terms of the unknown nodal heads or in terms of the unknown flow rate based on the continuity principle or work-energy principle. The equations of the water consumption and the head loss were formulated as nonlinear equations by Hamam and Brameller [24]. Though challenged by the nonlinear nature of the equation, the unique solution of this model is obtained on the basis of a prior knowledge of all fixed parameters and operating conditions. A real-coded genetic algorithm (GA) based model is developed in [25] for saving water and energy, by the use of the continuity equations and energy loss equations as the constraints of the optimisation problem. A modelling of a water distribution network to respond to the loop corrective flows and Newton-Raphson method numerical methods was developed in [24][25] and was referred to the loop simulator [26][27]. A hydraulic model was analysed in [28] based on the continuity equation and loss equation as well. In this method, the modelling of the network allows the computation of the friction coefficient based on the flow rate, the length, the diameter of the pipe. And the loop equations formulated in [29] are solved using the approach presented in [28] as a graph theory. The modelling equation was improved in [30] for a large network by including the coefficient of the fluid materials and considering the specifications of all the pipes that form the loop.

In this paper, a large water distribution network is modelled using the graph theory to improve reliability, efficiency and controllability of the model presented in [29][31]. This modelling formulates the network equations that can ensure an easy pressure control and can be further investigated and analysed to improve energy saving.

## III. Proposed Approach

### A. Equations modelling

Water distribution networks are represented by graph theory consisting of the finite number of nodes and pipes. Two main principles governed the network flow problems.

The first principle is the continuity or mass conservation equation at individual nodes. It is also known as Kirchhoff's first law: the algebraic summation of the flow at any given node is equal to zero. This is explained by:

$$\sum_{j}^{b} C_{ij} Q_j = -D_i, \forall i \tag{1}$$

where $C_{ij}$ is a matrix of n row and b columns node-branch matrix and

$$C_{ij} = \begin{cases} 1, & if\ branch\ j\ enters\ node\ i \\ -1, & if\ branch\ j\ leaves\ node\ i \\ 0, & Otherwise \end{cases} \tag{2}$$

$Q_j$ is the branch flow vector, *j* is the number of branches, *i* is the number of nodes and $D_i$ is the demand at the node *i*.

The second principle is the energy conservation equation for every loop of the network is described as the algebraic summation of the head losses of the pipes in the loop:

$$\sum_{j \epsilon L} h_j = 0 \tag{3}$$

With $h_j$ the head loss at the node *j* and *L* is the loop.

By considering the vector of coefficients of linearised, the head loss equation can be then written as

$$H = AQ \tag{4}$$

where *H* is the head loss and A is the vector of coefficient of linearised head-loss function.
A network is therefore described based on the head loss equation (Kirchhoff's second law) as [24]:

$$D\boldsymbol{\Delta P} = \Delta P_\beta \tag{5}$$

For a given branch $j$ and a loop $i$ of the network, $D_{ij}$ is equal to 1 if the branch $j$ is in the loop $i$ and is in the same direction, and $D_{ij}$ is equal to -1 if the branch $j$ is in the loop $i$ but in the same direction. If the branch $j$ is not in the loop $i$, $D_{ij}$ is equal to 0

The equations related to the pressure drop across the branches to the nodal pressures are

$$C^T P_\alpha = \Delta P_\beta \tag{6}$$

Where $P_\alpha$ is the nodal pressure, $\Delta P_\beta$ is the vector of pressure drop across the branches and $C^T$ is the transpose of $C$.

The hydraulic modelling for water distribution should include users water consumptions and generally this information is provided as an aggregated consumption during a period of time and the base demand of a node equation computed based on the sum of base demands consumers aggregated in this node and the total network consumption at the network inputs was formulated in [32] as:

$$D_i = \frac{bd_i}{\sum_{j=1}^{n_d} bd_j} q_{in} \tag{7}$$

where $D_i$ as stated above, is the demand at node $i$, $bd_i$ is the base demand of node $i$, $n_d$ is the number of the nodes in the network and is the total network consumption metered at sample at a given time.

The demand equation (7) may be improved by considering the daily variation of the relative pressure behaviour between two areas in the network and demand component [33].

The modelling of a water distribution network uses the continuity equations and the head loss equations to control the pressure around the node in the network. In this work the demand equation at each node as given in (7) under the assumptions that the behaviour of demand component of nodes in the same zone is similar, the aggregated consumption data are supplied for a short period of time and each node has its own demand based on different users that are connected to it.

The modelling in this work, evaluates based on the continuity principle, the amount of incoming water for specific period of time and compares this amount to the node demand in order to determine the control action of the pressure reduction valves.

The amount of incoming water through a node can be calculate as an algebraic sum of incoming flow rate for a given period of time and can be formulated as

$$B = t\sum_{j}^{b} Q_j \tag{8}$$

Where $B$ is the amount of incoming water at the node $i$ and $t$ is the delivery time period of time required.

The amount of incoming water required at the node for a period of time is compared to the water demand $D_i$ in order to decide on the operation of the pressure reduction valves.

$B_{ij}$ is the coefficient to evaluation in the required amount $B$ over a specific period of time. For a node $i$, $B_{ij}$ is equal to 0 if the amount of incoming water through the node meets the required amount in the acceptable limit of time, $B_{ij}$ is equal to 1 if the amount of incoming water through the node meets the required amount in a very short period of time (due to high pressure). $B_{ij}$ is equal to -1 if the amount of incoming water through the node does not meet the required amount in the acceptable limit of time (due to low pressure).

Although the node demands may vary from one node to another and for a period of time, a constant evaluation of the $B_{ij}$ for each node, may assist in determining the pressure control action to avoid leakages or pipes burst.

### B. *Graph based modelling*

The model of the water distribution network is based on the continuity equation and the head loss equation using graph theory.

Consider a network described by Fig. 1

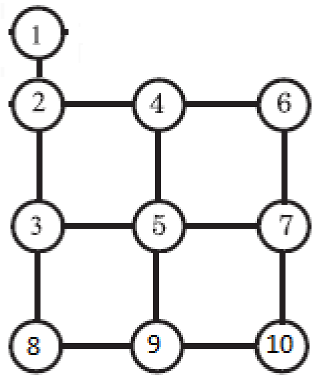


Fig. 1. Water distribution network for 10 node.

A spanning tree is determined in order to compute the initial flow distribution for the network. (1) and (3) provide the constraints to the network model ensuring that the two laws are satisfied.

With the network theory model which includes a finite number of nodes (junctions), finite number of edges (link or pipe) and the required pressure consumption knowledge at each node, the system graph that represent the connected configuration of the nodes and pipes is represented by the incident matrix. The incident matrix is used to ensure that incoming and outgoing flow rate at each node satisfies the required pressure at that particular node given that each node has different pressure requirement. In this paper, it is assumed that the pressure required at the node is determined based on the evaluation of the flow rate in all the pipes connected to the node and the demand in water at that node.

The pressure drop across a given branch to the flow in that branch equation (6) is necessary in determining the solution to the network. The modelling is very important in the determination of pressure

In the next section, the analysis and discussion a practical modelling network for pressure control is done.

## IV. Simulation and Discussions

The proposed approach for modelling of hydraulic water distribution network is simulated using data for a network using MATLAB software. The simulation is done for a DMA derived from Fig. 1 consisting of 5 nodes that are interconnected as shown by Fig. 2 below

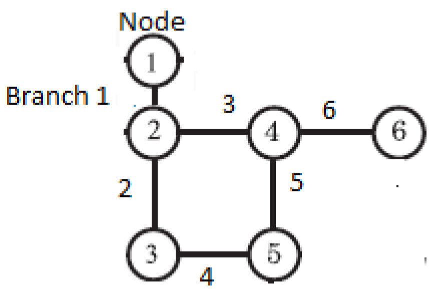


Fig. 2. Section of a DMA derived from the Water distribution network fig. 1 consisting of 6 nodes.

Table I below describes the behaviour of the nodes in the DMA illustrated by Fig. 2.

TABLE I. PIPES DESCRIPTION

| **Pipes number** | **Pipes elements** | | |
|---|---|---|---|
| | **From** | **To** | **Flow (liter/sec)** |
| 1 | 1 | 2 | 16 |
| 2 | 2 | 3 | 9 |
| 3 | 2 | 4 | 3 |
| 4 | 3 | 5 | 5 |
| 5 | 4 | 5 | 7 |
| 6 | 4 | 6 | 7 |

TABLE II. NODES DESCRIPTION

| **Nodes number** | **Nodes *Demands Indices* ($m^3$)** |
|---|---|
| 1 | 54 |
| 2 | 48 |
| 3 | 12 |
| 4 | 24 |
| 5 | 10 |
| 6 | 6 |

Table I describes the pipes connections between the node and the flow capacity. It is important to know that the pipe flow rate depend on the pump power and the pressure drop at the node of origin. The source pipe 1 requires more flow rate in order to supply enough quality water to the entire district meter area. Table II provides the water demands indices as measured at a given period of time. The water demands indicate the aggregated amount of water required at a given node. It is also important to identify that the source node 1, has the highest because it supplies water to the entire network.

To be able to determine the required pressure at the node graph theory to write the node-branch incident matrix of the district meter area as described in Fig. 1 with the nodes represented in the rows and the pipes are represented in columns.

$$C_{ij} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

The flow and the required power depend on network loop and for simplification our district meter area consists with only one loop with node 1 and node 6 not being part of the loop. The loop branch incidence matrix is given by:

$$D_{ij} = [0 \quad -1 \quad 1 \quad 1 \quad 1 \quad 0] \quad (11)$$

From the above matrices, the required pressure can be determined at a given node by considering the flow rate of the pipes that are connected to the node as well as the entire loop configuration in which the node is connected. The matrix $C$describes the relation between the nodes and the pipes. The required pressure at each node is calculated based on this matrix.

The node demands and the flow rate of the pipes that are connected to the nodes, given in the table I and II are therefore compare to ensure that the continuity and the energy conservation principles are observed at each node of the network. In the case the algebraic sum of the incoming water at is not equal to the outgoing amount water at a node, the entire network may experience a deficiency especially for the nodes connected in loop and creating leakages. Hence a pressure control mechanism is planned either to increase or decrease the flow rate with respect to the node demands. Fig. 3 below illustrates the flow rate (expressed in litre per second) relationship and this evaluation is identified in order to take an action on the pressure at the node.

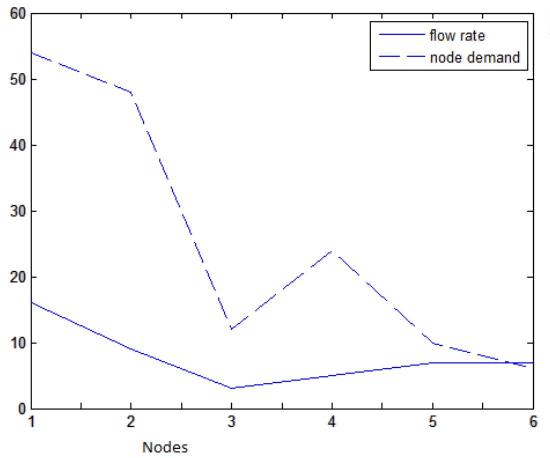


Fig. 3. Flow rate and node demands of the DMA

From the above figure 3, it may be observed that for the node demands of nodes 1,2,3,4 and 5 are above the flow rate at the node. This indicates that an amount of time is required for the nodes to meet the expected node demands. This can be improved by supplying more pressure at the source node 1 where the demand is very high compared to the flow rate during peak time. By doing so, node 2 receives more water per second and it will be able to distribute the required amount of water to node 3 and 4. For node 6, where the node demand and the flow rate are almost equal, every second the demand is met. In this case, if the usage of the water at this node is reduced, there is high probability of leakages to happen in the network. At such node, the installation of pressure reduction valves is very important to ensure efficiency and sustainability of the network.

It is important to note that with a good hydraulic modelling, the monitoring and control of pressure can be done easily since it helps to see the area that needs more focus such node 4 where the flow rate is very low compared to the demands at the node due to its strategic position in supplying node 5 and 6. At such node a pump might be required to increase the flow rate. Fig. 4 indicates the relationship between the required and expected flow rate (expressed in litre per second). The required flow rate is calculated based on the continuity equation to ensure a better functionality of the network and. And the pressure drop at the node is calculated accordingly in order to determine the required pressure based on the head loss and the above pressure drop equation.

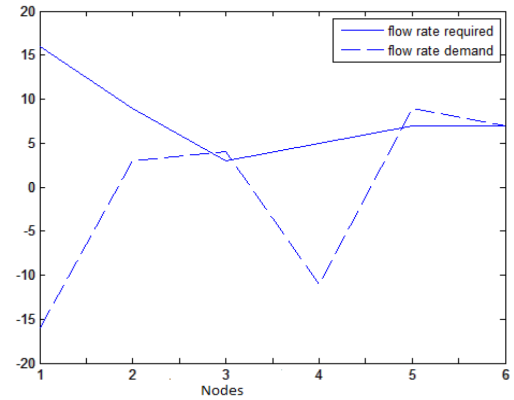


Fig. 4. Flow rate reqired vs flow rate demand of the DMA

## V. Conclusion

Pressure and flow regulating facilities are very important in water distribution networks to ensure the reliability and efficiency of water supply. The development and implementation of a robust and efficient solution to control the pressure is of importance and unavoidable for the sustainability of the network. This paper presents a hydraulic modelling approach to the pressure in water distribution networks. To be able to control and optimise the pressure in water distribution network, the adequate modelling is essential in the analysis of complex nodes and pipes network. This modelling evaluates pressure at each node based on the inflow and outflow rate. The pressure drop between nodes is examined to determine if the sum of the incoming flow will match the water demands and the outgoing flow at each node in the network. With such modelling, knowledge of the required pressure as well as the demand in water at each node, the water management system should be able to select the appropriate control methods easily since the simulation of a hydraulic modelling including pressure and flow control devices has more numerical challenges. This paper contributes to the modelling and analysis of water distribution network by explaining how the continuity and the energy conservation principles can be used to determine the appropriate pressure control decision in water distribution networks. This paper as well computes the amount of water required at a particular node for a period of time and compares it to the water demand in order to ensure sustainability and protect the pipes. The future works in this field will include different methods of control based on the relationship between the incoming water and outgoing water in order to provide the network with the required pressure at each node, mechanism of pressure control either by the use of pressure reduction valves or limiting the pumps power to avoid over supply of water than required. This paper elaborates clearly the specific pressure control decision on either to increase the pressure or to decrease.

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