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Efficient Synthesis of Large-Scale Thinned Arrays Using a Density-Taper Initialised Genetic Algorithm

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Abstract — **The use of the density-taper approach to initialise a genetic algorithm is shown to give excellent results in the synthesis of thinned arrays. This approach is shown to give better SLL values more consistently than using random values and difference sets for initialisation.**

1 INTRODUCTION

Thinned arrays have recently received significant attention in the literature due to their advantages over filled arrays. These include reduced cost and implementation complexity, and a narrower beamwidth for a given number of antenna elements. However, these benefits come at the cost of increased synthesis complexity.

Much of the thinned array literature has focused on the application of various optimisation algorithms to thinned-array synthesis including genetic algorithms (GAs) [1, 2], simulated annealing [3, 4], ant-colony optimisation [5], particle-swarm optimisation (PSO) [6], differential evolution [7] and the bees algorithm [8]. More recently, the iterative fast-Fourier-transform (FFT) technique (IFT) has been shown to produce excellent results even for extremely large arrays [9, 10]. The main drawback of these techniques is that they are iterative and require significant computation time to converge to good results.

Difference sets (DSs) of various types have recently received extensive consideration due to their unique properties which allow *a priori* bounds on the achievable sidelobe level (SLL) to be determined [11–16]. However, the relatively small number of known DSs [2] and the fact that the SLL values obtained using DSs alone are no better than those obtained using other techniques [16] limit the usefulness of DSs in thinned array synthesis.

In an attempt to overcome these problems, hybrid techniques that use DSs to initialise a GA (DS-GA) [2, 17] and a PSO (DS-PSO) [18] have been developed. This combination exploits the *a priori* information in DSs to both speed the convergence and improve the results of iterative optimisation algorithms. However, the limited number of known DSs remains a shortcoming of these hybrid approaches to the point that the generation of new DSs to initialise the hybrid algorithm has been recommended [2].

This work proposes the use of a density-taper (DT) algorithm to initialise a GA to overcome the limitations outlined above. A DT algorithm places elements in an aperture in such a way that the density of elements is proportional to the magnitude of some approximated excitation over the array aperture [19, 20]. While the principles underlying the DT approach are not new, they have recently attracted renewed interest due to their simplicity, generality and the excellent results obtained [9, 20].

2 ALGORITHM

Figure 1 gives a flowchart of the proposed approach whereby a DT algorithm is employed to initialise a GA (DT-GA). The DT and GA are described below.

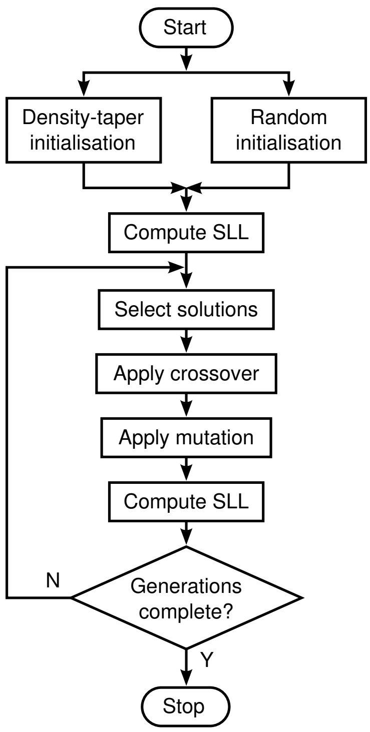

Figure 1: Flowchart of the proposed algorithm.

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2.1 Density-taper algorithm

The DT algorithm works by activating each element in a thinned array with a probability equal to the amplitude of the excitation at that element in a filled array [19]. It has been shown that the average power pattern of all possible thinned arrays synthesised in this way is given by [19]

$$\overline{|F(\theta,\phi)|^2} = |F_a(\theta,\phi)|^2 + \sum_{n=1}^{N_e} a_n(1-a_n) \tag{1}$$

where $\overline{|F(\theta,\phi)|^2}$ is the average power pattern, $F_a(\theta,\phi)$ is the pattern corresponding to excitations a_n and N_e is the number of elements. Usefully, equation (1) allows the average performance of the DT algorithm to be predicted *a priori.*

Chebyshev excitations [21] with random SLLs in the range -5 to -40 dB were used as the filled-array excitations. The probability that an element is active is thus equal to the Chebyshev excitation at that element. The SLL range was chosen to be significantly wider than the expected range of SLL values to ensure diverse solutions. The two-dimensional excitations were obtained by using independent Chebyshev excitations with the same SLL along the two axes of the array [21].

The DT approach to initialising other algorithms has the benefits that good results are obtained and a large number of diverse solutions can be generated. The second point is especially important during initialisation because diversity must be maintained to ensure that the complete solution space is searched [22].

2.2 Genetic algorithm

The GA implemented here is similar to that proposed by [1,2], but with four differences highlighted below.

Firstly, a specified portion (including 0) of the population is initialised using the DT algorithm or a DS. Random initialisation is used for the remainder of the population.

Secondly, uniform crossover is used instead of single-point crossover [22]. Uniform crossover means that each bit of a child is randomly selected from one of its two parents. Crossover probability was 1.0 and mutation probability was 0.005.

The third difference is that the best individual from each generation is copied to the next generation unchanged (elitism) to ensure that the best solution is never lost [22].

Lastly, exponential ranking selection is used because it allows control of the selection intensity (how strongly better solutions are favoured) [23]. Solutions are selected according to

$$\frac{P_{n-1}}{P_n} = \frac{P_n}{P_{n+1}} = c < 1 \tag{2}$$

where P_n is the probability of selecting solution n and better solutions have higher values of n. The parameter $\kappa = c^{N_P}$ where N_P is the population size decouples the selection intensity from the population size [23] and $\kappa = 0.14$ was used here.

3 RESULTS

Results for both linear and planar arrays are summarised in Table 1 and the implications of these results are considered below. The linear case is considered in more detail because run times are shorter.

Case	Runs	SLL (dB) Best	Median	Worst
200-element linear array				
DT	10,000	-22.49	-18.54	-13.24
GA	100	-22.00	-20.48	-19.13
DS-GA	100	-23.44	-22.27	-20.53
DT-GA	100	-23.94	-23.35	-22.39
24×24-element planar array				
DT	10,000	-20.92	-16.64	-0.02
GA	100	-21.32	-20.43	-19.60
DS-GA	100	-21.25	-20.52	-19.75
DT-GA	100	-22.53	-21.62	-20.73

Table 1: Summary of results. The GA results use 20 individuals and are run for 100 generations.

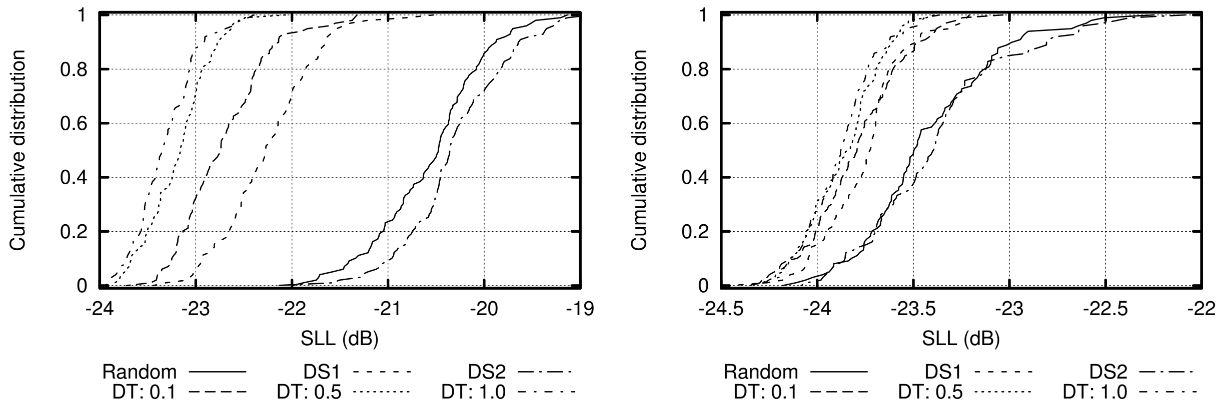
3.1 Linear arrays

A 200-element linear thinned array (≈3 m long at 10 GHz) is used as the linear test case. While much of the literature enforces symmetry to reduce the number of variables (e.g. [1, 8]), this was not done here in agreement with [2, 17].

All pattern computations were performed using a 2048-point FFT during optimisation and a 8192-point FFT for the final results.

The DT algorithm alone exceeds the best DS result in more than 78% of 10,000 tests clearly demonstrating the superiority of the DT algorithm in this application.

The results from 100 runs of the GA are summarised in Figure 2 by plotting cumulative distributions. Results are shown for random initialisation, and DT and DS initialisation. The DS initialisation was performed using all the known cyclic DSs and almost DSs with lengths from 190 to 200 [24, 25] and results for the best two DSs are reported.



(a) 20 individuals run for 100 generations.

(b) 50 individuals run for 200 generations.

Figure 2: Results obtained by initialising a GA in different ways. The values are the proportion of the population initialised using the specified technique. DS1 and DS2 denote the 197-147-109-98 and 190-95-47-142 almost difference sets respectively.

The first important observation is that the best results obtained using DS initialisation (DS1) are significantly better than using random initialisation. However, the second-best DS-based results (DS2) are actually worse than random initialisation, and the remaining 22 DSs considered are worse still. The importance of the availability of a suitable DS is thus clearly demonstrated.

The effect of DT initialisation is even more dramatic, and even initialising only 10% of the population with a DT algorithm produces noticeably better results than the best DS-based initialisation. Increasing the proportion of the population initialised leads to still better results, though the rate of improvement decreases above 50%. As stated above, this improvement is due to the excellent results obtained and the diverse solutions generated with the DT algorithm.

Comparing Figures 2(a) and 2(b) shows that increasing the population size and number of generations decreases the differences between the various initialisation schemes, while the same general trends are observed. Though not shown, results for all approaches converge when the population size and/or number of generations are increased sufficiently. This occurs because the GA explores the problem more thoroughly and thus becomes less sensitive to initialisation.

3.2 Planar arrays

A 24×24 array (≈1.44×1.44 m at 10 GHz) was used as the planar test case. While this array is not as large as some cases considered in the literature (e.g. [9]), it does contain 576 elements and is the largest size for which square DSs are available [12].

All pattern computations were performed using a 512×512-point FFT during optimisation and a 2048×2048-point FFT for the final results.

The DT algorithm exceeds the best DS result [12] on more than 55% of 10,000 runs, again clearly demonstrating the superiority of the DT approach in this application.

The GA results in Table 1 are noticeably better than the DS and DS-PSO techniques results published previously [12, 18]. The DS-GA results are very similar to the GA results suggesting that there is no major benefit to the use of DS initialisation in this case. As for linear arrays, this is probably a consequence of the limited number of known DSs. The DT-GA results are again significantly better than the other cases considered, with an improvement of over 1 dB over the GA and DS-GA results. As before, this is due to the quality and diversity of solutions generated by the DT approach.

4 CONCLUSION

The use of a DT algorithm to initialise a GA for the synthesis of thinned arrays is considered. This approach is shown to reliably produce excellent SLL values with minimal additional complexity.

The DT algorithm alone produces better results than the best DS results for the problems considered. Furthermore, the results obtained using the DT algorithm to initialise a GA are better to those obtained when random values and DSs are used for initialisation.

There is every reason to believe that similar improvements will be obtained when DT algorithms are used to initialise other iterative synthesis techniques, thereby greatly enhancing these algorithms' ability to synthesise large arrays without significantly increasing their complexity.

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