Pressure management of water distribution systems via the remote real-time control of variable speed pumps

Philip R. Page a, Adnan M. Abu-Mahfouz b,c, Matome L. Mothetha a

a Built Environment, Council for Scientific and Industrial Research (CSIR), Pretoria, 0184, South Africa
b Department of Electrical Engineering, Tshwane University of Technology, Pretoria, 0001, South Africa
c Meraka Institute, Council for Scientific and Industrial Research (CSIR), Pretoria, 0184, South Africa

Corresponding author email address: pagepr7@gmail.com

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Abstract

Low and constant pressure can be maintained throughout a water distribution system by setting the pressure at remote consumer locations and using the pressure to control the speed of a variable speed pump (VSP). The prospect of incorporating hydraulics theory into a controller is investigated, with the goal of improving on generic controllers. Five new controllers are proposed here, four of which depend on hydraulics theory. These controllers, which set the speed of the VSP, are investigated numerically. A parameter-dependent controller that does not require the flow in the pump to be known is developed and shown to significantly improve on the performance of conventional (parameter-dependent) proportional control (PC). Next, a parameter-free controller that requires the flow in the pump to be known is proposed and shown to outperform PC, even though PC has a tunable parameter, and perform comparably to the best new parameter-dependent controllers. The parameter-dependent controllers (when optimally tuned) perform best overall. The efficacy of many of the new controllers shows that hydraulics theory can lead to improved controllers.

Keywords: Water distribution system; Pressure management; Remote real-time control; Variable speed pump; Hydraulic modelling

1 Introduction

The pressure in a water distribution system (WDS) can be adjusted via the use of pressure control valves (PCVs), turbines, pumps-as-turbines (PATs) and variable speed pumps (VSPs), in response to real-time pressure sensor measurements at various remote nodes [1]. In remote real-time control (RRTC) the pressures at these nodes are set to be low and constant [2, 3]. A conventional or variable speed pump’s purpose is to impart energy to the water (the direct mode), while a PAT is a pump which operates in reverse mode.

Pumps can be divided into either positive displacement and rotodynamic pumps. The latter is considered here. A rotodynamic pump is a dynamic device where the head is generated by a rotating impeller [4]. In this work the focus is on VSPs. In a VSP the angular velocity (rotational speed) \( \omega \) of the pump impeller can change (in contrast to conventional pumps). The main benefits of a VSP are [4]: (1) Energy savings: Savings of between 30% and 50% have been achieved by installing VSPs in place of conventional pumps; (2) Improved control: Small variations in the WDS pressure can be corrected rapidly, because the speed can be finely tuned. This leads to less likelihood of flow or pressure surges; (3) Improved pump reliability: Reduction of pump wear, particularly in bearings and seals; and reduction of noise and vibration. Nevertheless, VSPs are not always the best design solution in terms of long term costs [5].

This work concerns the control of VSPs to set the pressures at remote nodes to be low and constant, while ensuring that the pressures are high enough for consumers. While not discussed in detail here, there are two expected benefits: (1) A low pressure in the WDS will decrease the leakage in pipes, damage of pipes and consumption [1]. (2) The VSPs supply the lowest heads needed for ensuring that the pressures are high enough for consumers, which tends to be associated with energy savings [6].

Keeping the average node pressure in the WDS constant as water consumption changes, by changing the speed of the VSP, has recently been studied [7]. A hydraulic model for a South African rural village WDS was considered, with water being pumped from a reservoir into a tank feeding a reticulation network. This forms part of a larger study [8, 9].

By contrast, the common solution to pressure management in the literature, considered from now on, is to seek to keep the pressure constant at specific nodes. In a first approach to keeping the pressure constant, an accurate detailed hydraulic model of the real-world WDS is assumed to be available. Here general computationally efficient mathematical techniques have been developed to solve exactly for the VSP speed inside a hydraulic model if one node pressure is fixed to a certain value, in a wide variety of scenarios [10, 11]. In a second approach, programmable logic controllers are used to keep the pressure constant in a real-world WDS. This does not require the construction of a hydraulic model. This work studies the latter approach by modelling the operation of these controllers with the help of a hydraulic model of the WDS, assuming that the model is a good surrogate for the real operation of the WDS.

The current research on VSPs is in the context of the RRTC paradigm of work on PCVs originally performed on proportional control [2, 12]; and later extended to control where the flow in the PCV needs to be known [3, 13].

Pressure management was demonstrated using five VSPs at a WDS located in Poznań (Poland) [6]. However, the control uses statistical procedures, and is not RRTC. Various generic RRTC controllers, which do not use any hydraulics theory, have been used in experimental laboratory test-beds that model a WDS [14, 15, 16, 17]. These include an extension of proportional integral derivative (PID) control to active disturbance rejection control for modulating one VSP [14]. This methodology enhances the performance of a classical feedback controller, like the PID, by adding a disturbance observer that reconstructs and rejects the unwanted perturbation in the system in each control cycle [14]. The following two methods were used to develop controllers, which were used to modulate one PCV and one VSP: (1) Fuzzy logic, which is an attempt at the formalisation/mechanisation of the human capability to make rational decisions [15]; (2) Generalised minimum variance self-tuning, where the parameters were estimated in real-time with a recursive least squares method [16, 17].

Five new RRTC controllers are proposed in this work, four of which are derived from hydraulics theory and are investigated numerically for an example WDS. The Conclusion section highlights the findings.

2 Pump control

A pump imparts a positive head \( \bar{H} \) to the liquid, often called the “total dynamic head” or “pump lift” [18], and here called the pump head. This is the amount of energy increase per unit weight of the liquid imparted to the liquid by the pump. A VSP can be used to maintain a set pressure value at a (remote) control node of the WDS. The VSP changes the pressure to the required setting by varying \( \omega \). In particular, a pump increases/reduces the pump head in order to increase/reduce the pressure at the control node to the set-point. The pump manufacturer provides the information of how \( \bar{H} \) depends on \( Q \), which is the pump discharge flow rate (at the pump outlet). Particularly, a curve \( \bar{H} = g(Q) \) is provided, where \( Q_{\text{min}} \leq Q \leq Q_{\text{max}} \). Here \( Q_{\text{min}} \geq 0 \), and \( \bar{H} \geq 0 \) for the allowed range of \( Q \). The curve is specified for rated conditions \( \omega = \omega_R \), where \( R \) denotes “rated”. The curve is called the pump characteristics or pump head capacity curve [18], and here simply the pump curve.

The affinity laws for head and flow rate can be used to obtain the relationship between \( \bar{H} \) and \( Q \) for any value of \( \omega \). These laws apply to rotary implements: both to centrifugal and axial flows in rotodynamic pumps. The laws are accurate because they are based on actual tests for all types of
centrifugal pumps, and the errors are extremely small [18]. The pump curve is
\[ \tilde{H} = \alpha^2 g \left( \frac{Q}{\alpha} \right) \text{ for } Q_{\text{min}} \leq \frac{Q}{\alpha} \leq Q_{\text{max}} \text{ where } \alpha \equiv \frac{\omega}{\omega_R} \]  
(1)

This defines the dimension-less pump speed \( \alpha \) (same notation as [18]), where \( \alpha > 0 \). It has a maximal value, with \( \alpha = 1 \) at the rated conditions. Typically the pump only operates in the range \( \omega \leq \omega_R \), so that the maximum value of \( \alpha \) is unity.

A WDS node which is sensitive (defined below), and as far as possible, also has the lowest pressure [2], is called a critical node (CN). Pressure management in a WDS with one pump will be accomplished by attempting to keep the pressure constant at a specific CN.

The following controller is proposed which adjusts the pump head, a hydraulic variable, in analogy to the “head-loss” controller for PCVs [3]. It is suspected that this controller involves much more complicated pump speed adjustment than the simple smooth adjustment required for controllers that set \( \alpha \), because it adjusts a hydraulic variable. There is a change in the head at the control node \( H \), given by the differential relationship \( dH = \frac{d\tilde{H}}{S} \), as \( \tilde{H} \) is changed. Here \( S \) is the sensitivity, a function of \( \tilde{H} \) at a certain state of the WDS.

The “head-gain” controller is
\[ \tilde{H}_{i+1} = \tilde{H}_i - S_i (H_i - H_{sp}) \]  
(2)
where \( H_{sp} \) is the target set-point head of the control node. The next iteration \( i + 1 \) is determined by iteration \( i \), which are separated by a control time-step \( T_c \). As in [13], the “head-gain” controller is taken, for theoretical considerations, to be the superior controller. As is shown the Appendix, one reason for the superiority is that the controller can be derived from the Newton-Raphson numerical method. Define \( S_i \equiv S(\tilde{H}_i) \), which has different values for different iterations. When the CN head depends very sensitively on the pump head, the value of \( S_i \) is called the ideal sensitivity (\( S_i = 1 \) for a pump). If ideal sensitivity is used in Eq. 2, there are then no adjustable quantities in the “head-gain” controller, i.e. it is parameter-less.

At each time-step \( T_c \) the pump speed is changed. Unsteady flow processes [2, 3, 13] and poor convergence of the controller [3] can be limited by restricting the rate of speed change to a maximum velocity \( \nu_\pi \). This is defined as the maximum change in \( \omega \) in a time interval \( \Delta t \), divided by \( \omega_R \Delta t \).

Convergence of the controller limits the value of \( \nu_\pi \) that should be used in the controller, even if the physical pump allows a larger value [19].

3 Controllers which set the pump speed

This section describes controllers which set the pump speed \( \alpha \), a physical property of the pump.

The proportional control” method (denoted PC), in analogy to the method for PCVs [2, 3, 12, 13], adjusts
\[ \alpha_{i+1} = \alpha_i - k_\pi (H_i - H_{sp}) \]  
(3)
where \( k_\pi \) is a dimension-full parameter.

Now endeavour to derive controllers which are not just generic, but employ theoretical understanding of the hydraulics of a WDS. For rated conditions, the pump curve is \( \tilde{H} = g(x) \), where the function \( g \) is evaluated at \( x = Q \). Define \( g'(x) \) to be the derivative of \( g \) with respect to \( x \) evaluated at \( x \). For application in what follows, \( g' \) should explicitly be calculated from \( g \).

From Eq. 1 the differential relationship
\[ d\alpha = \frac{1}{\alpha} h_1(x) d\tilde{H} + h_2(x) dQ \text{ where } x = \frac{Q}{\alpha} \]  
(4)
is obtained (see Appendix), with
\[ h_1(x) \equiv \frac{1}{h(x)} \quad h_2(x) \equiv -\frac{g'(x)}{h(x)} \quad \text{where } h(x) \equiv 2g(x) - xg'(x) \]  
(5)
Consider the derivation of controllers from Eq. 4 where the \( dQ \) term is neglected. This hence assumes that \( Q \) remains constant from iteration to iteration. Using Eq. 4 and the “head-gain” controller in Eq. 2, the controller

\[
\alpha_{i+1} = \alpha_i - \frac{1}{\alpha_i} h_1(x_i) S_i (H_i - H_{sp})
\]

is proposed. This controller is usually not practical for use in a real-world WDS because the sensitivity \( S_i \) is required to be calculated from a hydraulic model of the WDS. Generally, control of a pump only makes sense by using a sensitive node to set the pressure; thus the requirement to set it at a CN. In this work, a sensible controller is constructed by substituting the ideal sensitivity in Eq. 6. This is because \( S_i \) may not be known; and the equation becomes (1) manifestly independent of the WDS (although the pump curve is still used) and (2) parameter-less. The controller in Eq. 6 is accordingly called the “parameter-less P-controller with known constant pump flow” (LCF). The use of ideal sensitivity is in analogy to the “valve resistance” control for a PCV [3].

The flow does not remain constant for most WDSs. In an attempt to correct for this and incorporate the missing dynamics, \( S_i \) can be set to the dimensionless parameter (proportional constant) \( K_\pi \) in Eq. 6. An analogous procedure was followed for a PCV [13]. The proposed controller is called the “parameter-dependent P-controller with known constant pump flow” (DCF).

The LCF and DCF controllers require \( Q \) to be known, either through a field measurement [3, 13] or a hydraulic model prediction of \( Q \). Installing a flow meter at the site of the pump would incur additional financial cost. These controllers offer robustness with respect to changing WDS conditions, because they depend on demand through their dependence on \( Q \). This is exemplified by the fact that for the analogous controller to DCF for PCVs, the parameter changes only slightly when the daily average demand change in the year [13]. If \( Q \) is not known, the following controller is proposed from Eq. 6

\[
\alpha_{i+1} = \alpha_i - \frac{k'_\pi}{\alpha_i} (H_i - H_{sp})
\]

which does not depend on \( Q \). Here \( k'_\pi \) is a dimension-full parameter. This controller effectively assumes that \( h_1 \) is constant. However, the controller can be used even if this is not the case. The controller in Eq. 7 is called “pump proportional control modified” (denoted PCM). It is closely related to PC in Eq. 3. The PC and PCM controllers offer limited robustness with respect to changing WDS conditions.

Now keep the \( dQ \) term in Eq. 4. From the “head-gain” controller in Eq. 2, a new controller is proposed, called the “parameter-less controller with known variable pump flow” (LVF)

\[
\alpha_{i+1} = \alpha_i - \frac{1}{\alpha_i} h_1(x_i) S_i (H_i - H_{sp}) + h_2(x_i) \Delta Q_i
\]

where \( S_i \) is set to unity to obtain a parameter-less controller. Here, \( dQ \) is approximated by \( \Delta Q_i \) as defined in the Appendix. All other controllers analysed in this work are based on calculating the change in the pump speed, which is proportional to the difference between the head at the CN and the set-point head (called \( P \)-control). This is not true for the LVF controller.

Using the inequality in Eq. 1, Eqs. 6 and 8 assume

\[
Q_{\text{min}} \leq x_i \leq Q_{\text{max}} \quad \text{where} \quad x_i = \frac{Q_i}{\alpha_i}
\]

LCF, DCF and LVF offer robustness with respect to changing WDS conditions. They explicitly depend on the pump curve, and hence assume that the pump curve and the affinity laws are accurate. Interestingly, the pump curve is not restricted to a specific shape.

To decrease the effect of unsteady flow processes, \( H_i \) can be measured by adopting a pressure moving average [12]. The same technique can be used to measure the pump flow.

The value of research into controllers that depend on the sensitivity (Eqs. 2, 6 and 8), which become parameter-less with ideal sensitivity, partially lies in the identification of the preferable algorithmic form, because of the lack of freedom to introduce arbitrary parameters. Improved efficacy
beyond a controller which depend on sensitivity can always be attained by proposing a new parameter-dependent controller, through adding tunable parameters to the original controller. In this work, the focus is on testing the algorithmic form of LVF, so that no tunable parameters are added in Eq. 8.

4 Speedtown WDS

The simulation of RRTC pressure management using VSPs is possible with available software packages, although no published details of their use for pumps have been found. These are the “WDNetXL pressure control module” [3, 20]; and EPANET On-Line, which uses PID control [21]. The package used here is a newly programmed extension of the one discussed in [19] to also include VSP modulation. It interacts with an extended-period hydraulic solver, so that the controller can be validated on a hydraulic model of a WDS. The demand factor time-variation is read at intervals $T_c$.

In the numerical simulations use $T_c = 5$ min (in accordance with [3, 12, 13]); and $\nu = 0.0002 \text{s}^{-1}$, which yields a maximum change in $s$ of $\nu T_c = 0.06$ in one time-step.

In order to allow a degree of comparison with the study of PCVs and PATs, the example WDS is based on that of Jowitt and Xu [22], specifically as implemented in [23] (see Figure 1). The WDS is, amongst others, used in the studies [2, 12, 24]. Leakage is implemented according to [23]. In addition, the effect of pressure-dependent demand is taken into account. The demand factor varies substantially between 0.6 and 1.4 (see Figure 2) [22]. This drives the change of $Q$ over time.

Since the Jowitt and Xu WDS does not contain a pump, the WDS is minimally modified and called “Speedtown”, after pump “speed”. Water no longer gravitates down from three reservoirs, but is pumped from a single distant reservoir at high altitude to the remainder of the WDS at low altitude. The reservoirs at nodes 23-25 that were previously at very similar water levels are replaced with simple pipe junctions each at the same elevation 15 m. The new pipes from these nodes to node 26 are each 1 km long with a diameter of 457 mm (the largest pipe diameter size in the Jowitt and Xu WDS). Water is pumped from a distant reservoir via a 30 km main pipe with a diameter 1.5 times 457 mm. All Hazen-Williams major friction coefficients of newly added pipes are 100. None of the newly added junctions have leakage. Node 26 is at elevation 15 m, and the reservoir and node 27 at elevation 45 m. Moreover, all nodes in the original Jowitt and Xu WDS with elevations larger
than 15 m are set to have an elevation of 15 m. Node 22 at elevation 15 m consistently has the lowest pressure. The sensitivity $S_i$ of this node when pressure control is performed is in the range 1.131 to 1.233, with an average value of 1.184. Hence node 22 is sensitive, so that it is chosen to be the remote pressure control CN. The target set-point pressure head $p_{sp}$ at the CN is taken to be 30 m (also used in [23]). $S_i$ is assumed to be unity for all applications of parameter-less controllers to the example WDS.

The static head at the CN, which is the difference between the head of the reservoir and the CN, is zero when the set-point pressure is reached. This has the consequence that when the pump is not pumping, i.e. $Q = 0$, the VSP operates at $H = 0$. It is generally desirable for a VSP to induce a small static head [4].

The pump curve is assumed to have the physically reasonable form $\tilde{H} = c - aQ^2$, with $a$ and $c$ positive constants, and $b = 2$. The power needed by the pump to lift the liquid, per unit weight of the liquid, is $Q\tilde{H}/\eta$ [18]; where the efficiency $\eta$ is assumed to be constant in a relevant flow-head region for this WDS. A calculation yields that this is maximised when $\tilde{H} = 2aQ^2$. One point which satisfies this is the operating point $Q = \tilde{H} = 0$ previously mentioned. Another point is chosen to the operating point for the highest demand (demand factor 1.4), representing the rated conditions, which lies both on the pump curve and maximal power curve. Fitting to the total demand when the target pressure is reached at the CN yields $a = 331.6 \text{ s}^2/\text{m}^5$ and $c = 54.67 \text{ m}$. Hence at least two of the operating points of the pump (those at $\alpha = 1$ and at $\tilde{H} = Q = 0$) have maximal pump power, ensuring that the pump operates near to maximal power for other speeds.

It is found that $\nu_\pi$ as low as 0.00004 s$^{-1}$ can be used in Speedtown without changing any of the results. This corresponds to the time taken for the pump to change from full to zero speed being 6.9 hours, emphasizing that unsteady flow processes are likely limited.

### 5 Efficacy of controllers

Let $\Delta$, according to Eq. 2 of [13], be the temporal average of the absolute value of the difference between the head at the control node and $H_{sp}$. $\Delta$ measures the difference between the control node pressures and the set-point pressure, i.e. how well the pressure is controlled. Let $\delta$ be the maximum over time of the absolute value of the difference between the head at the control node and $H_{sp}$ [19]. Hence the mean and maximum deviations are measured by $\Delta$ and $\delta$ respectively. These are small.
Table 1: Efficacy of controllers. † Same values as for DCF.

<table>
<thead>
<tr>
<th>Controller</th>
<th>Eq.</th>
<th>Parameter</th>
<th>$\Delta$ (m)</th>
<th>$\delta$ (m)</th>
<th>$\langle \Delta \alpha \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional PC</td>
<td>3</td>
<td>$k_\pi = 0.015$ m$^{-1}$</td>
<td>0.0261</td>
<td>0.101</td>
<td>0.00055</td>
</tr>
<tr>
<td>PCM</td>
<td>7</td>
<td>$k'_\pi = 0.0118$ m$^{-1}$</td>
<td>†</td>
<td>†</td>
<td>†</td>
</tr>
<tr>
<td>Known $Q$ DCF</td>
<td>6</td>
<td>$K_\pi = 1.29$</td>
<td>0.0068</td>
<td>0.024</td>
<td>-</td>
</tr>
<tr>
<td>Known $Q$ LCF</td>
<td>6</td>
<td>Parameter-less</td>
<td>0.0674</td>
<td>0.204</td>
<td>0.00105</td>
</tr>
<tr>
<td>Known $Q$ LVF</td>
<td>8</td>
<td>Parameter-less</td>
<td>0.0128</td>
<td>0.056</td>
<td>0.00018</td>
</tr>
</tbody>
</table>

for Speedtown (Table 1). For a controller with a tunable parameter, the case with minimal $\Delta$ and $\delta$ is indicated, which is referred to as the **optimally converged** case. Such a controller is optimised for the specific demand variation assumed in Figure 2, and will not perform as well (without extra retuning) for different conditions [14] measured by using smart water infrastructure [25, 26, 27]. It is hence unsurprising that these controllers will in general outperform parameter-less ones when only one demand condition is considered.

From Table 1 the most optimally converged controller is DCF. The time-dependence of the pump speed can be studied for the various controllers. Particularly, the temporal average of the absolute value of the difference between the pump speed for a controller and DCF, denoted $\langle \Delta \alpha \rangle$ (indicated in Table 1), is a measure of the mean deviation of the pump speed of a controller from the most optimally converged controller. The time-dependence of the pump speed for the various controllers is almost identical.

For DCF, $\Delta$ is within a factor of two of its minimal value for $K_\pi$ in the range 1.22 to 1.36, and the functional dependence of $\Delta$ on $K_\pi$ is fairly flat. The controller should operate in this **effective range** of $K_\pi$ [12]. Since optimal convergence is usually not obtained in the real-world, the parameter of a parameter-dependent controller can, as a rule of thumb, be considered to be anywhere in the effective range for comparison purposes to other controllers.

For the pump curve used in Speedtown, $h_1(x) = 1/(2c)$, a constant, so that the PCM and DCF controllers will give identical results, as indicated in Table 1. It is significant that PCM, which is closely related to the conventional PC, yields 3.8 times better control for Speedtown (as deduced from Table 1 by comparing $\Delta$’s).

The variable flow parameter-less LVF improves by a factor of 5.3 on the constant flow LCF. These parameter-less controllers are compared in Figure 3. The ratio of the last term to the second last term of LVF in Eq. 8 is found to always be positive and nearly constant as a function of time. The time-averaged ratio is 0.36. Hence, the controller is effectively similar in character to DCF with $K_\pi = 1.36$, which can be seen by comparing to Eq. 6. LVF is compared to the optimally converged parameter-dependent controllers in Figure 4. Remarkably, the value of $\Delta$ for LVF is within those obtained for parameters in the effective range for DCF, even though only the latter controller has the luxury of a tunable parameter. Moreover, LVF outperforms the optimally converged PC by a factor of 2.0.

For the parameter-less controllers in the example WDS, the times with the smallest pressure deviations in Figure 3 approximately coincide with the times when the demand in Figure 2, and hence $Q$, changes the slowest.

### 6 Conclusion

The novel “head-gain”, PCM, LCF, DCF and LVF controllers are proposed. The latter four employ theoretical understanding of the hydraulics of a WDS, set the speed of the VSP, and are investigated in detail for a specific demand variation. For parameter-dependent controllers where the flow needs not be known, with limited robustness with respect to changing WDS conditions, the PCM shows 3.8 times improvement over the conventional proportional control (PC) for Speedtown. Also, for all controllers investigated, the time-dependence of the pump speed is similar.

For LCF, DCF and LVF the pump flow needs to be known, offering robustness with respect to changing WDS conditions [13]. The parameter-less LVF is 5.3 times better than the parameter-
less LCF. As expected, DCF, which is extension of the parameter-less LCF to include a tunable parameter, improves LCF (by a factor of 9.9 for the optimally converged DCF). Remarkably, LVF compares well to all controllers with tunable parameters: It improves on the optimally converged PC by a factor of 2.0; and is within the effective range for the top-performing PCM and DCF.

Control using fuzzy logic and generalised minimum variance self-tuning, which has tunable parameters, resulted in a maximum error on pressure control of 3.11–3.47% [15] and 2.12% [17], and an
average error of 1.02% [15], when applied to an experimental test-bed containing a PCV and VSP. By contrast, the maximum error for the parameter-less LVF ($\delta/p_{sp}$), for example, is 0.19%, and the average error ($\Delta/p_{sp}$) 0.04%, although these errors do not include various real-world complications.

The excellent performance of various proposed controllers make them viable for adoption in real-world environments. However, their performance has only been verified for a single VSP, and when there are no tanks. A limitation of the controllers where the pump flow needs to be known, is that the pump curve must be known accurately.

A Appendix: Details of “head-gain” controller, pump curve and LVF

The Newton-Raphson numerical method has as its goal to find $z$ such that $f(z) = 0$, i.e. find the root of a function of one variable. $z$ is found by the iteration [28]

$$z_{i+1} = z_i - \frac{f(z_i)}{f'(z_i)} \tag{10}$$

Identifying $z$ with $\bar{H}$ and defining $f(\bar{H}) = H(\bar{H}) - H_{sp}$, means the goal is to find $\bar{H}$ such that $H(\bar{H}) = H_{sp}$, as required. Applying Eq. 10 leads to the “head-gain” controller in Eq. 2. This derivation assumes that $f$ depends on only one variable, which is, for example, not correct when demands and reservoir levels change over time.

The LCF, DCF and LVF controllers require the pump curve $g$ to be differentiable (see Eqs. 5, 6 and 8). Also, these equations show that the controllers can only be well-defined if $g(x) \equiv g'(x)$. This will almost always be true since $g(x) \geq 0$ and $g(x) \geq 0$ in the allowed range, and it is almost always the case that $g'(x) \leq 0$ in the allowed range (see Appendix B of [29] for example pump curves).

Some development and most notation in this Appendix are similar to those for PCVs, and are hence further clarified in the Appendix of [30]. At time $t_{i-1}$ the pump speed and flow are $\alpha_{i-1}$ and $Q_{i-1}$ respectively. For controllers which rely on setting $\alpha$, the adjustment process starts soon after, continuing up to time $t_{i-1} + T_{c(i-1)}$. Now the pump is completely adjusted to the new speed $\alpha_i$, and the flow is $\bar{Q}_i$. At time $t_i \equiv t_{i-1} + T_{c}$ the speed is still $\alpha_i$; and the pump flow $Q_i$.

A feature of Eq. 1 that is important for what follows is that it expresses $\bar{H}$ in terms of variables that does not include $H$. Moreover, $\bar{H}$ can be calculated once the independent variables $\alpha$ and $Q$ are specified. The analogous equation for PCVs, which expresses the head-loss across the PCV in terms of only the head-loss coefficient and flow rate variables [3, 12, 13], has been used to develop new controllers [3, 13].

By differentiating $\bar{H}$ in Eq. 1, $d\bar{H}$ can be written in terms of $\alpha, Q, d\alpha$ and $dQ$, where $d\alpha$ and $dQ$ are independent changes. This leads to Eq. 4. $dQ$ is approximated by $\Delta Q_i$ for the LVF controller in Eq. 8. Using $\Delta Q_i \equiv Q_i - \bar{Q}_i$, was as was done for the numerical simulations, does not involve $\alpha$ changing. Using $\Delta Q_i \equiv Q_i - Q_{i-1}$, which involves $\alpha$ changing, yields inferior results, with $\Delta = 0.0359$ m and $\delta = 0.113$ m.

As noted in the Appendix of [30], $dQ$ can be estimated (1) by a demand prediction algorithm, or (2) when it is not the case that $t_c(i-1) \ll T_c$, by

$$\Delta Q_i \equiv (Q_i - \bar{Q}_i) \frac{T_c}{T_c - t_c(i-1)} \tag{11}$$

References


11