Random wave fields and scintillated beams

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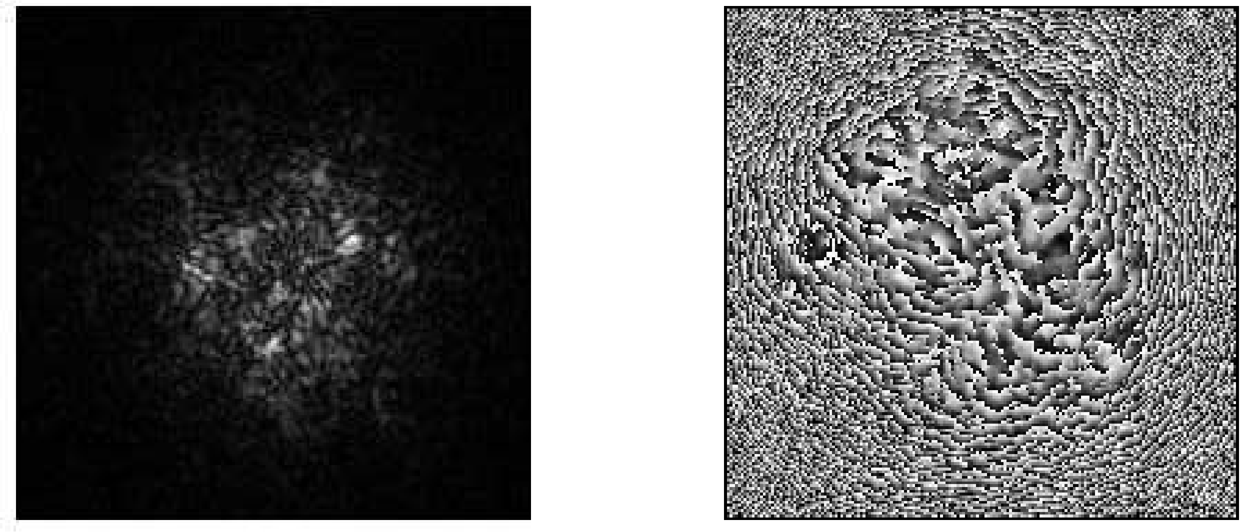
Scintillated optical beams

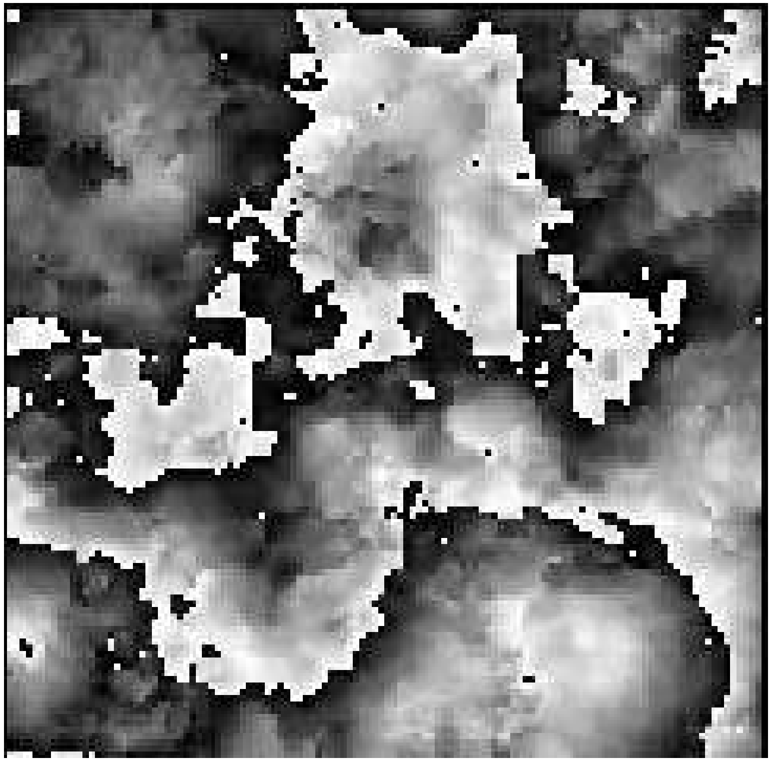
When an optical beam propagates through a turbulent atmosphere, the index variations cause random phase modulations that lead to distortions of the optical beam.



Weak scintillation

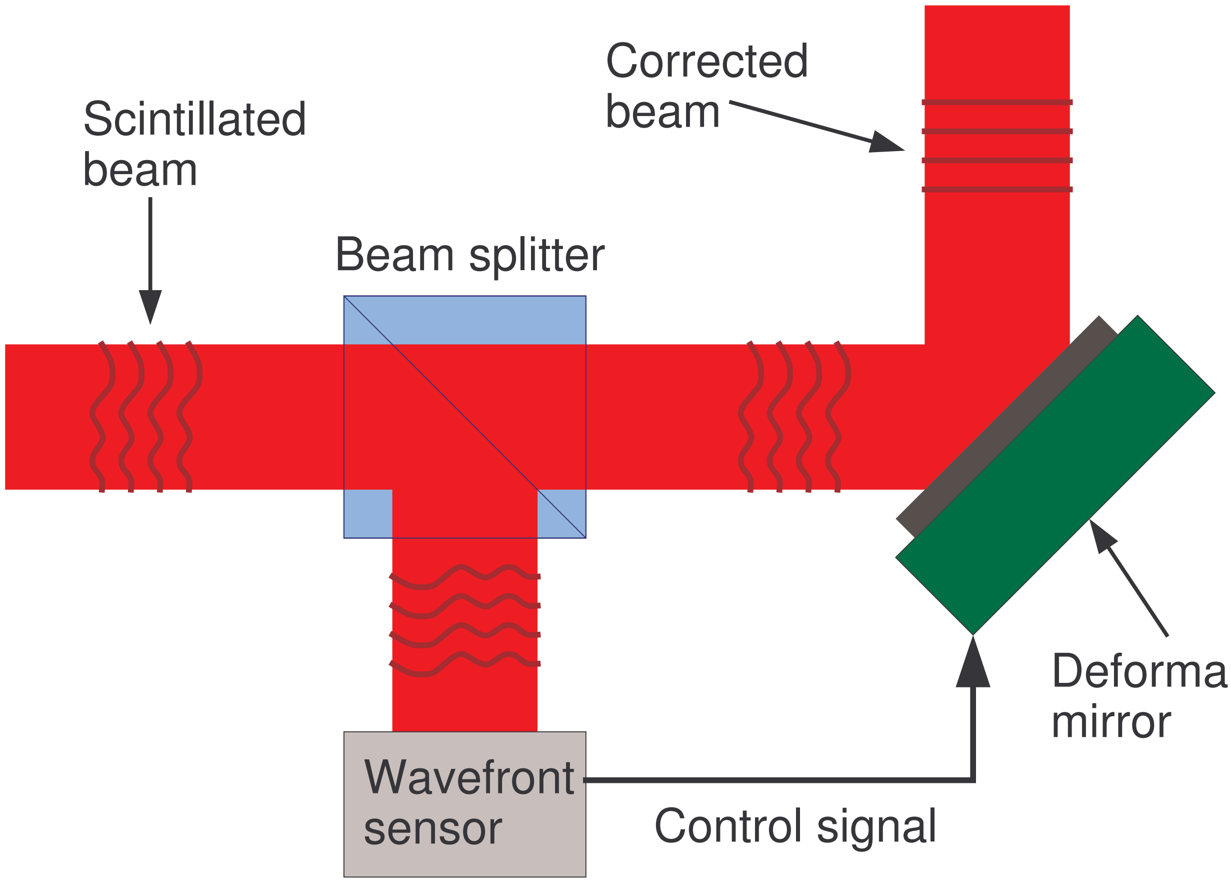
If the scintillation is weak the resulting phase function of the optical beam is still continuous.

Such a weakly scintillated beam can be corrected by an adaptive optical system.



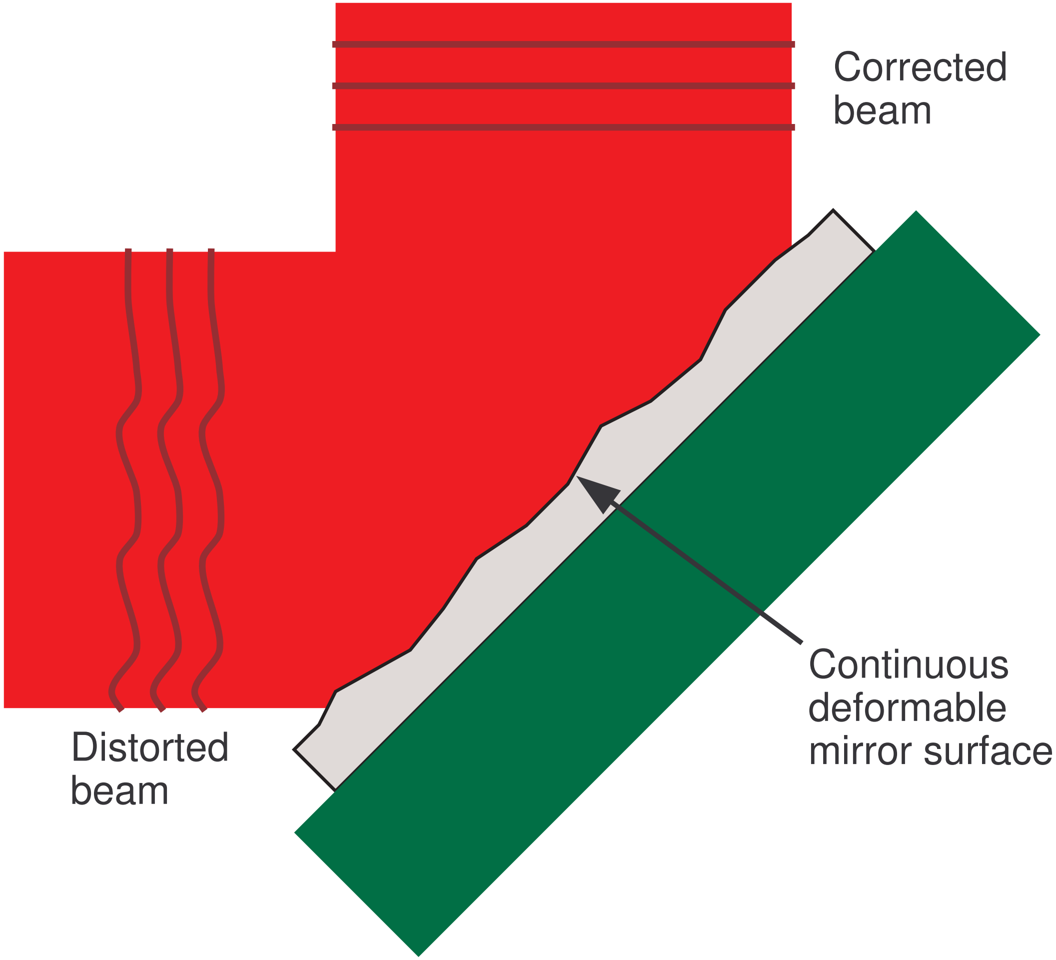
Adaptive optics

An adaptive optical system measures the continuous phase distortions in an optical beam and uses a deformable mirror to correct the distortions.



Deformable mirror

A deformable mirror can correct phase distortions by changing the shape of the surface of a deformable mirror.



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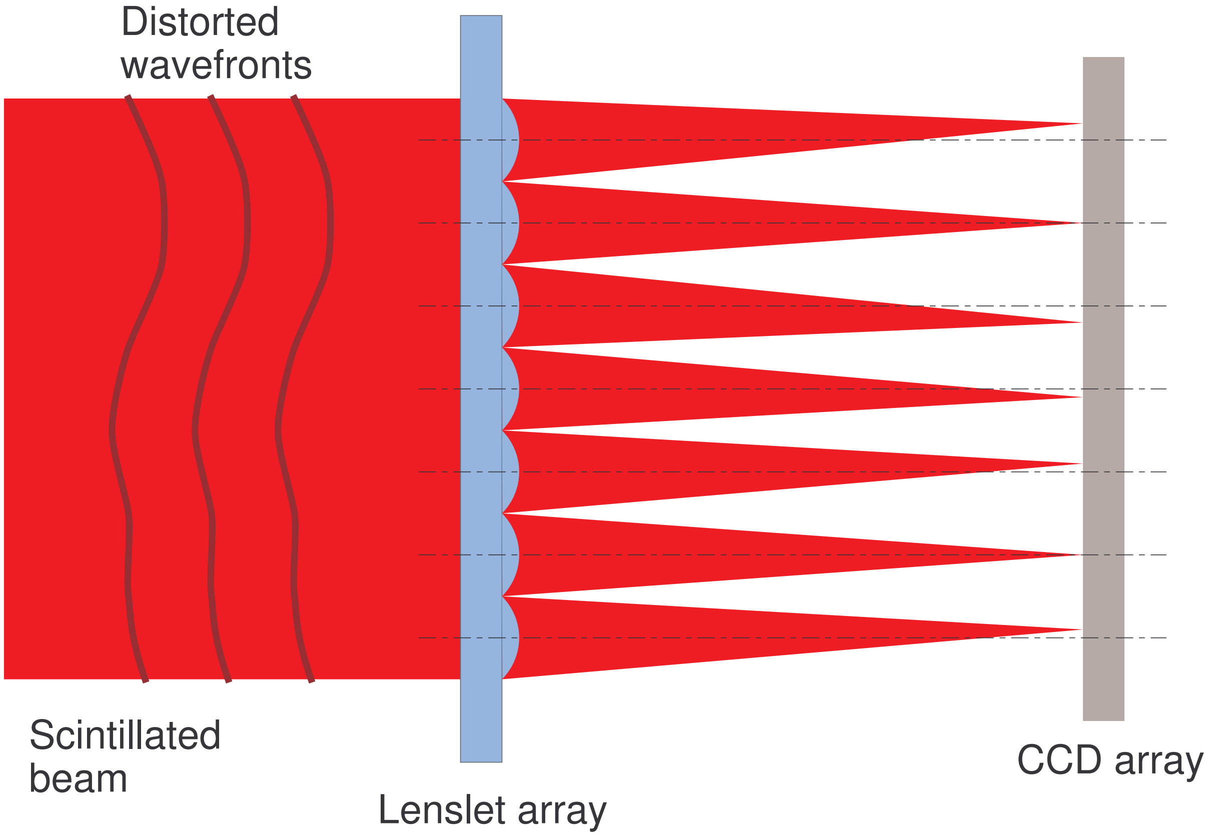
Shack-Hartmann wavefront sensor

A wavefront sensor is used to detect the phase distortions that are present in the beam. The Shack-Hartmann wavefront sensor consists of a lenslet array and a CCD array. It measures the gradient of the phase function.



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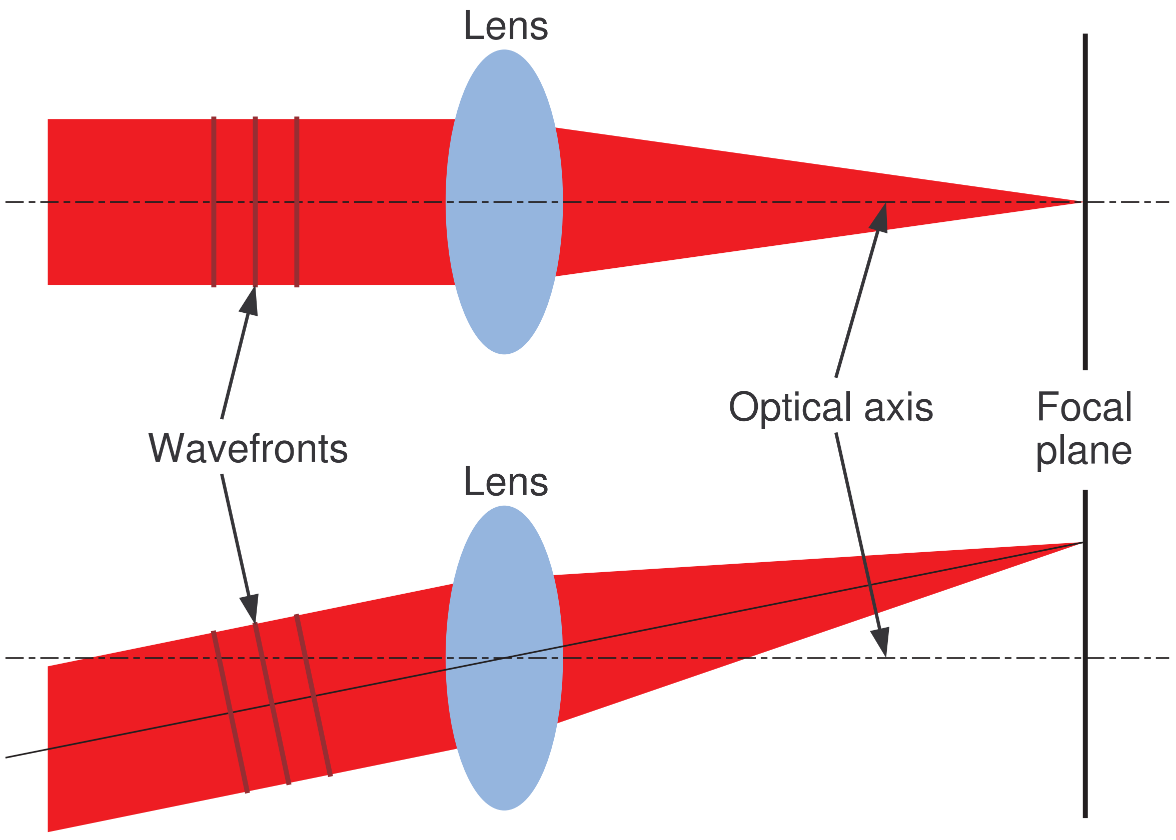
Phase gradient measurement

The local gradient in the phase function is determined by the offset of the focal point caused by the tilt in the wavefront.



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Least-squares algorithm

- ▷ Measured phase gradients → compute required adjustments → deformable mirror
- ▷ Least-squares estimate of phase from measured phase gradients
- ▷ Computation: linear (vector-matrix multiplication)

$$\mathbf{P}\theta = \Delta\theta$$

$$\Rightarrow \theta_{LS} = \left(\mathbf{P}^T\mathbf{P}\right)^{-1}\mathbf{P}^T\Delta\theta$$

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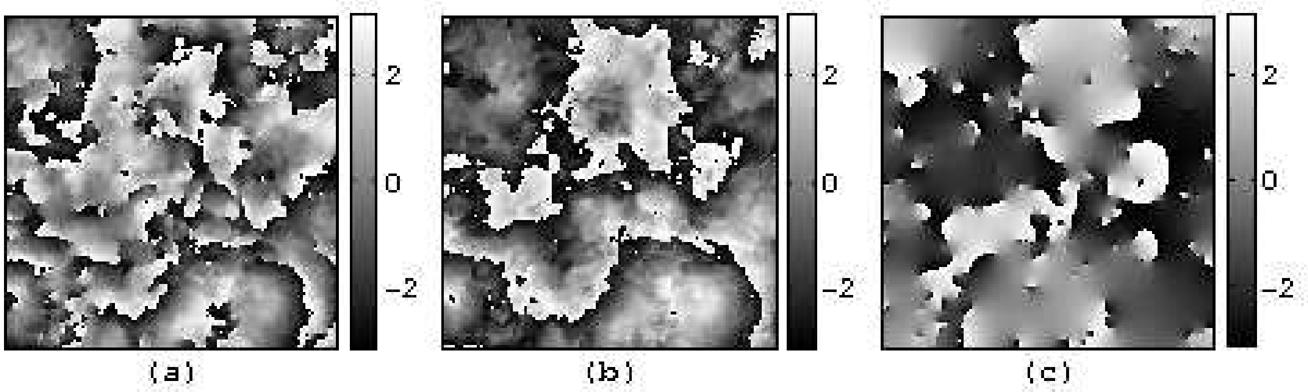
Strong scintillation

When the scintillation becomes strong enough, the phase distortions become severe enough to form optical vortices. Then conventional adaptive optics does not work anymore.



Detecting optical vortices

Adaptive optics cannot directly remove optical vortices because they are phase singularities that cannot be cancelled off by a deformable mirror. It is necessary to remove the vortices first. In order to remove them one first need to locate them.

One can use the output from a Shack-Hartmann sensor to detect optical vortices.

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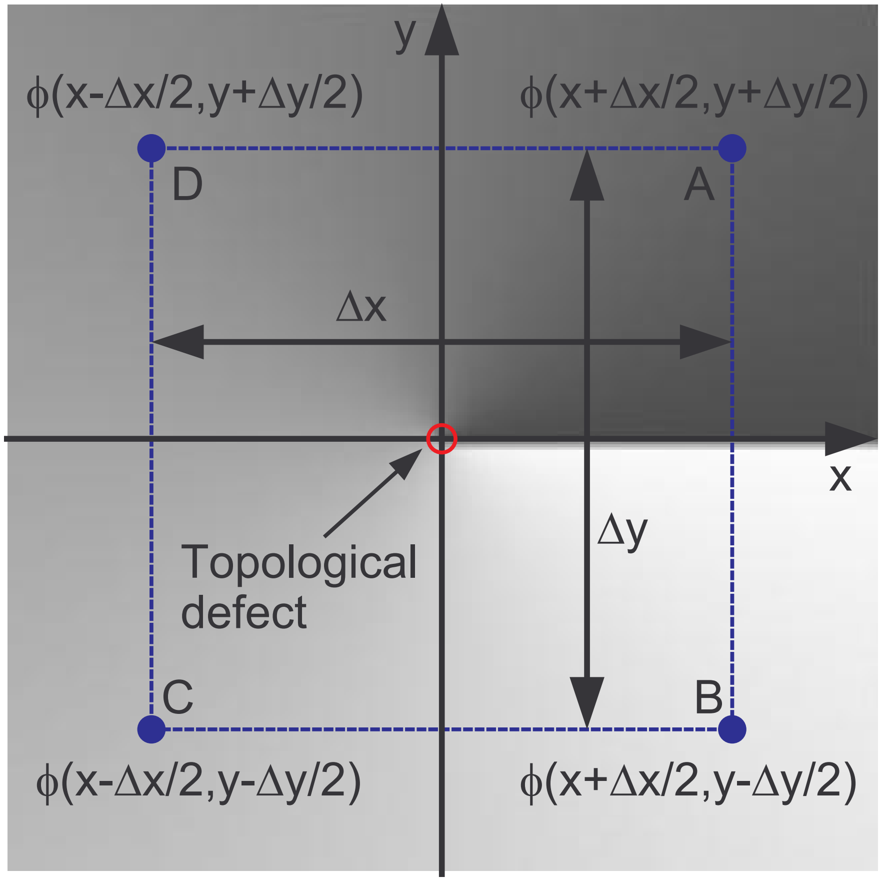
Curl of the gradient

It can be shown that for single vortex

$$\nabla \times \nabla\phi = 2\pi\delta(x, y)$$



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Vortex density

$$\oint_C \nabla\theta \cdot \hat{\mathrm{d}}s = \iint_A \nabla \times \nabla\theta \cdot \hat{\mathrm{d}}a = \nu\ 2\pi$$

$$(\nabla \times \nabla\theta) \cdot \hat{z}\ \mathrm{d}a = \mathrm{d}\nu\ 2\pi$$

Topological charge density:

$$D(x,y) = \frac{\mathrm{d}\nu}{\mathrm{d}a} = \frac{1}{2\pi}\left[\nabla \times \nabla\theta(x,y)\right] \cdot \hat{z}$$

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Detecting a vortex with SH

- ▷ Shack-Hartmann wavefront sensor: measures local phase gradient $\mathbf{F}$ from shifts in focal points behind lenslet array.
  $\Rightarrow$ Locate optical vortices with curl: $D = \nabla \times \mathbf{F}$
- ▷ Theoretical value is 2π. Actual value smaller than 2π due to finite aperture size.
- ▷ Use threshold to select vortex out of noise.

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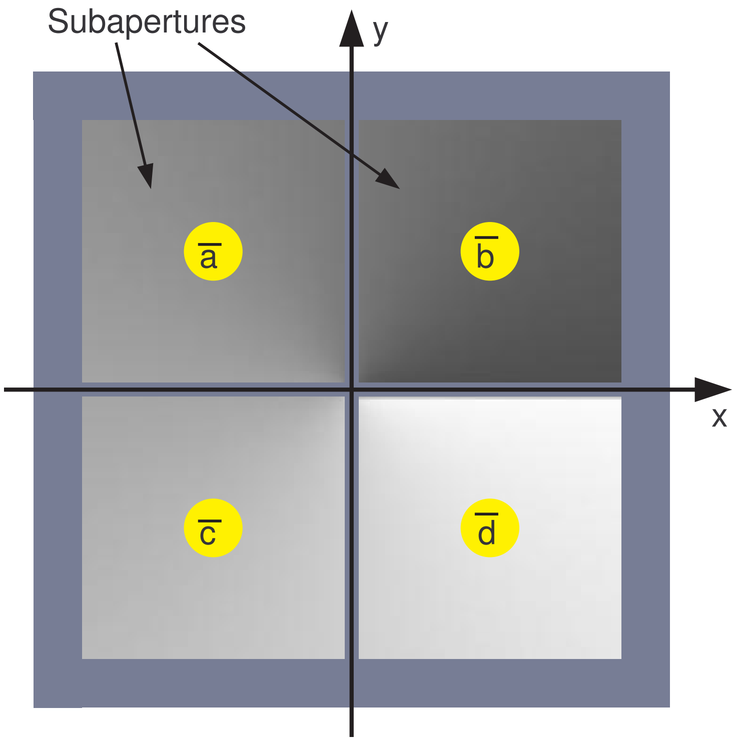
Circulation

Discrete output from Shack-Hartmann wavefront sensor → compute circulation (discrete version of topological charge density)

$$D = \frac{1}{2}\left[(b_y - a_y) + (d_y - c_y) - (a_x - c_x) - (b_x - d_x)\right]$$



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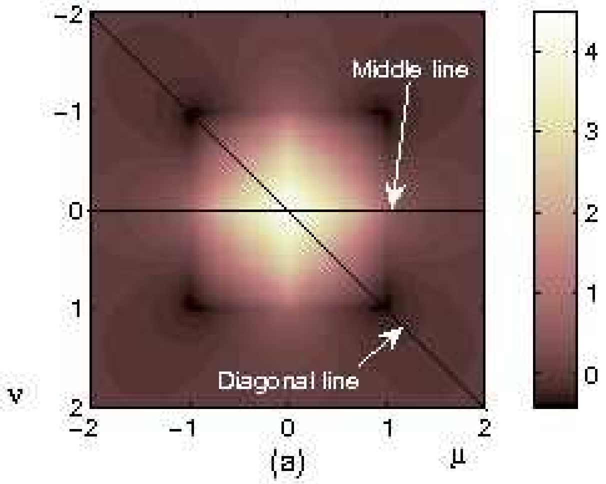
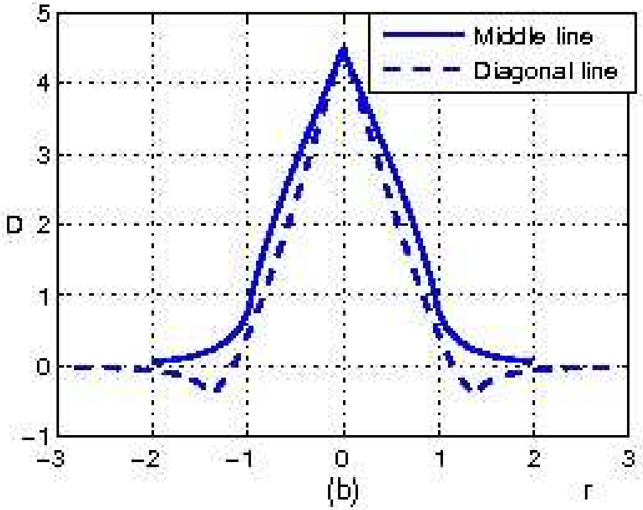
Effect of shift

Peak value: $\pi + 2\ln(2) = 4.53$
If vortex is not located
in the centre among four
apertures, the circulation
values is smaller.



(a)



(b)

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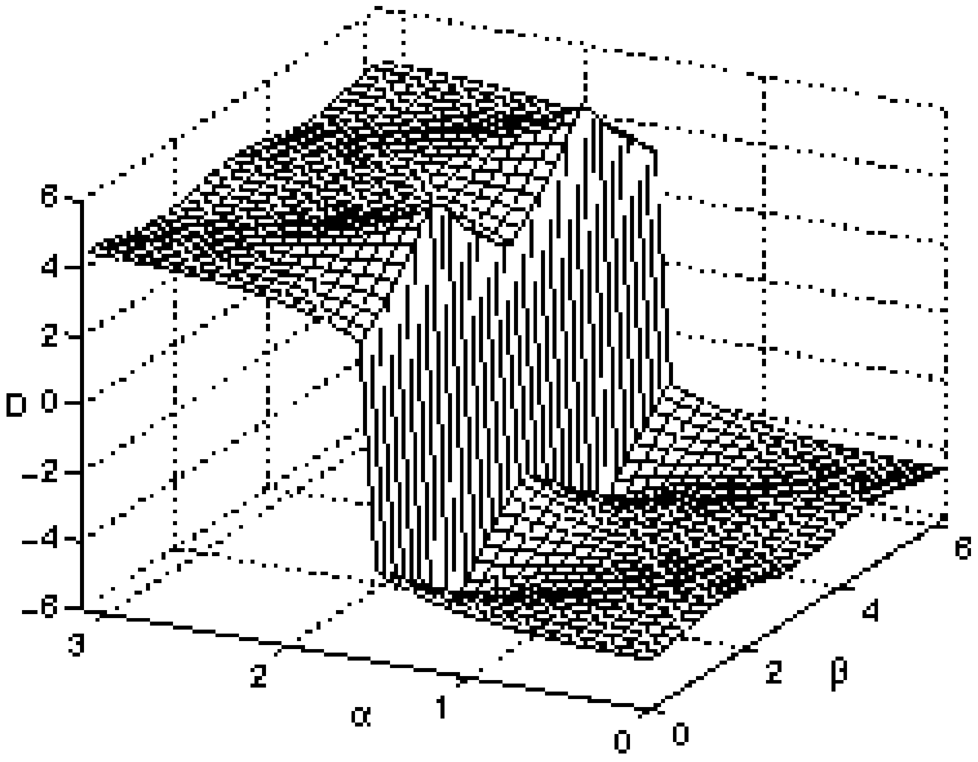
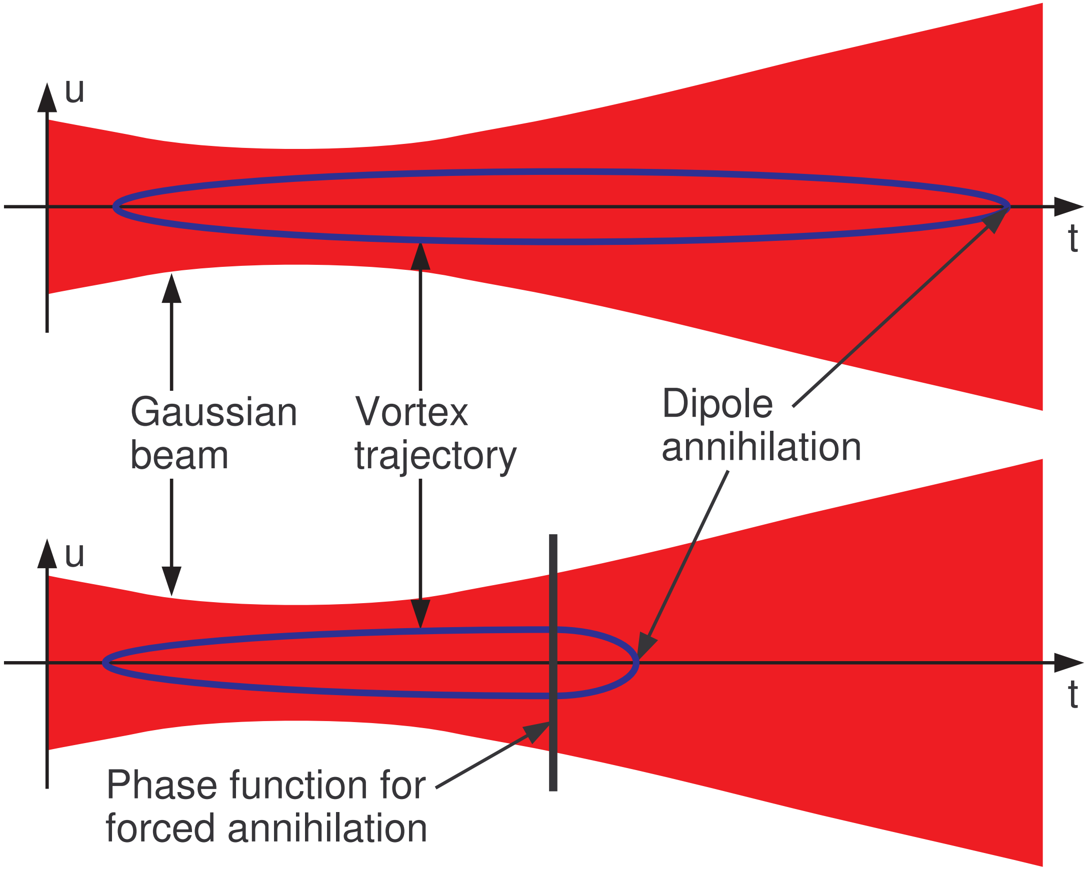
Effect of morphology

For vortices with different morphologies the circulation values fluctuates, but differs significantly for two topological charges.



Forces annihilation

To get rid of optical vortices in strongly scintillated optical beams, the idea is to force a vortex dipole to annihilate sooner by introducing a special phase function.



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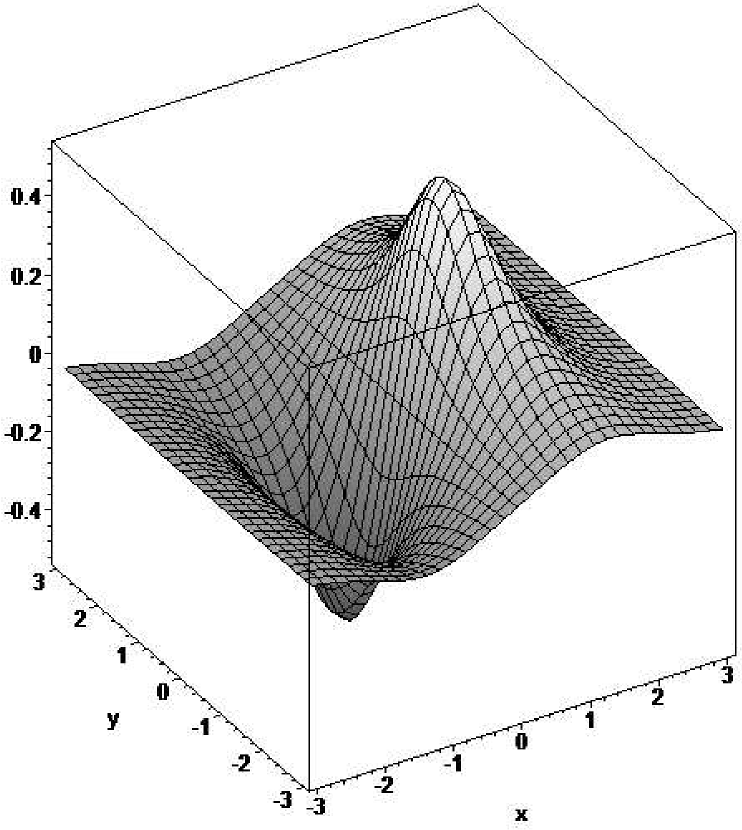
Annihilation in Gaussian beam

Using our knowledge of vortex propagation in Gaussian beams, we use phase function at the annihilation point. The idea is that this phase function still contains the power to force a vortex dipole to annihilate.



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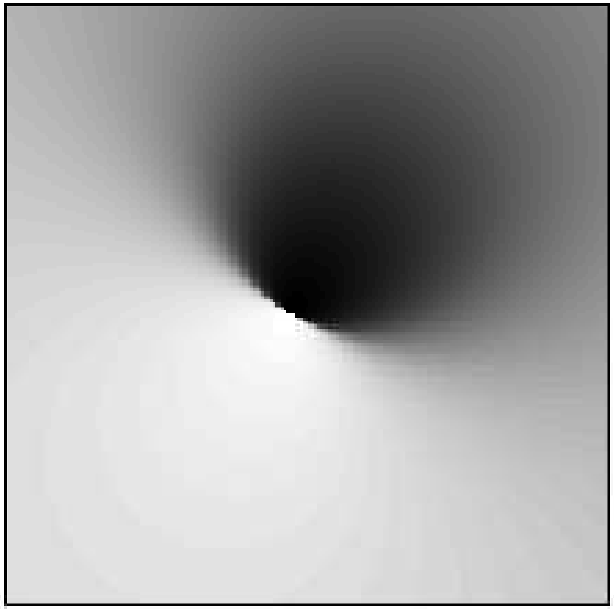
Vortex removal procedure

The procedure to remove optical vortices is then as follows:

- ▷ Located all the optical vortices.
- ▷ Divide them into dipoles.
- ▷ Compute annihilation phase function for each dipole.
- ▷ Multiply beam with all these phase functions.
- ▷ Allow beam to propagate.



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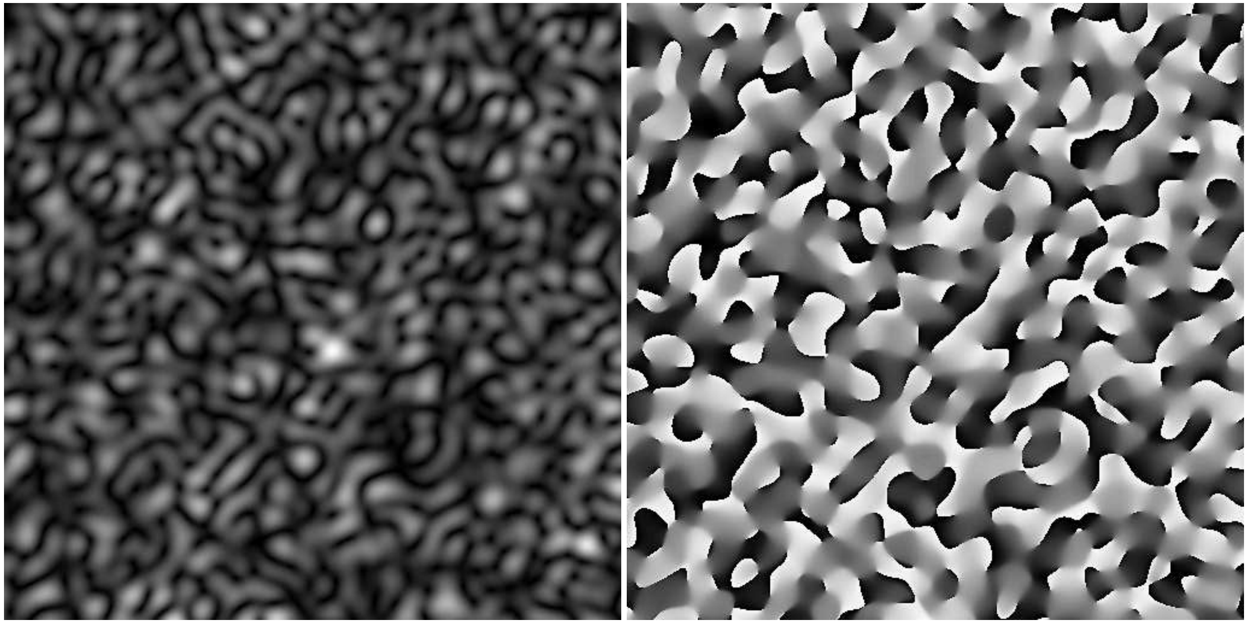
Random vortex fields

A random wave field (speckle field) is also a random vortex fields.



Amplitude Phase

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Random wave field

One can generate a random wave field by summing a number of plane wave with random complex amplitudes

$$f(x,y,z)=\sum_n \alpha_n \exp\left(i\mathbf{k}_n\cdot\mathbf{x}\right)$$

where $\mathbf{k}_n$ are propagation vectors all lying within a certain cone angle θ, which determines the coherence area of the field:

$$\text{Radius of coherence area}=\frac{\text{wavelength}}{\sin\theta}$$

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Properties of random vortex fields

Random vortex fields have the folowing properties:

- ▷ Vortex density inversely proportional to coherence area.
- ▷ Globally tend to neutral topological charge.
- ▷ Adjacent topological charge is anti-correlated.
- ▷ During propagation: annihilation rate = creation rate $\Rightarrow$ equilibrium

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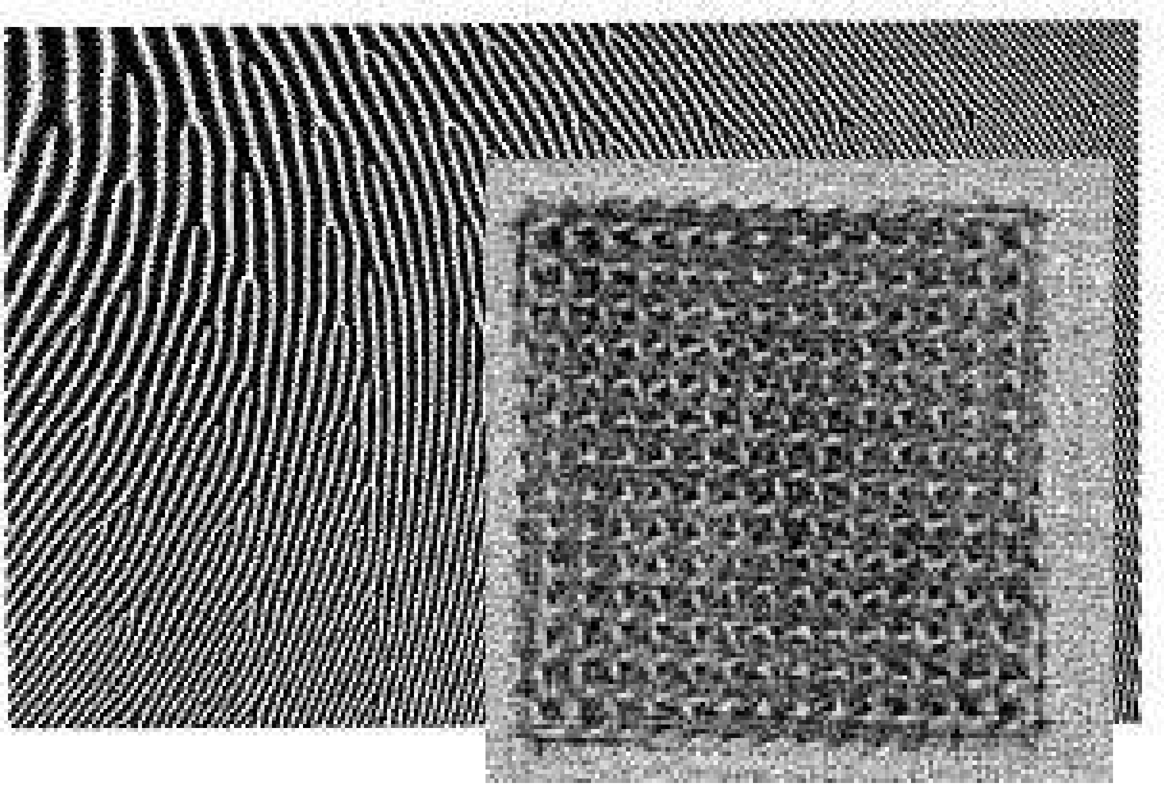
Geometrical transforms

Rotation transform



⇒ artificial vortex field

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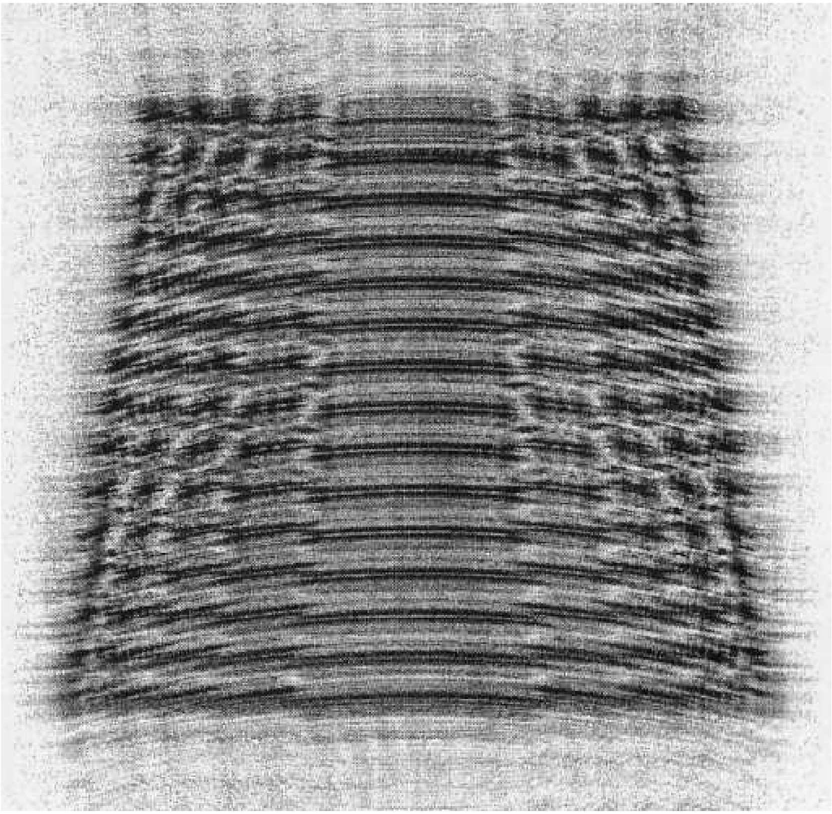
Polar formatting transform

Correcting distortion of SAR data.



⇒ another artificial vortex field

Artificial vortex fields

- Artificial vortex fields can be generated by CGH (geometrical transforms) or SLM.
- Separate vortices with opposite topological charge into different regions.
- What happens during propagation?
  Equilibrium will be restored.

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Density limitations

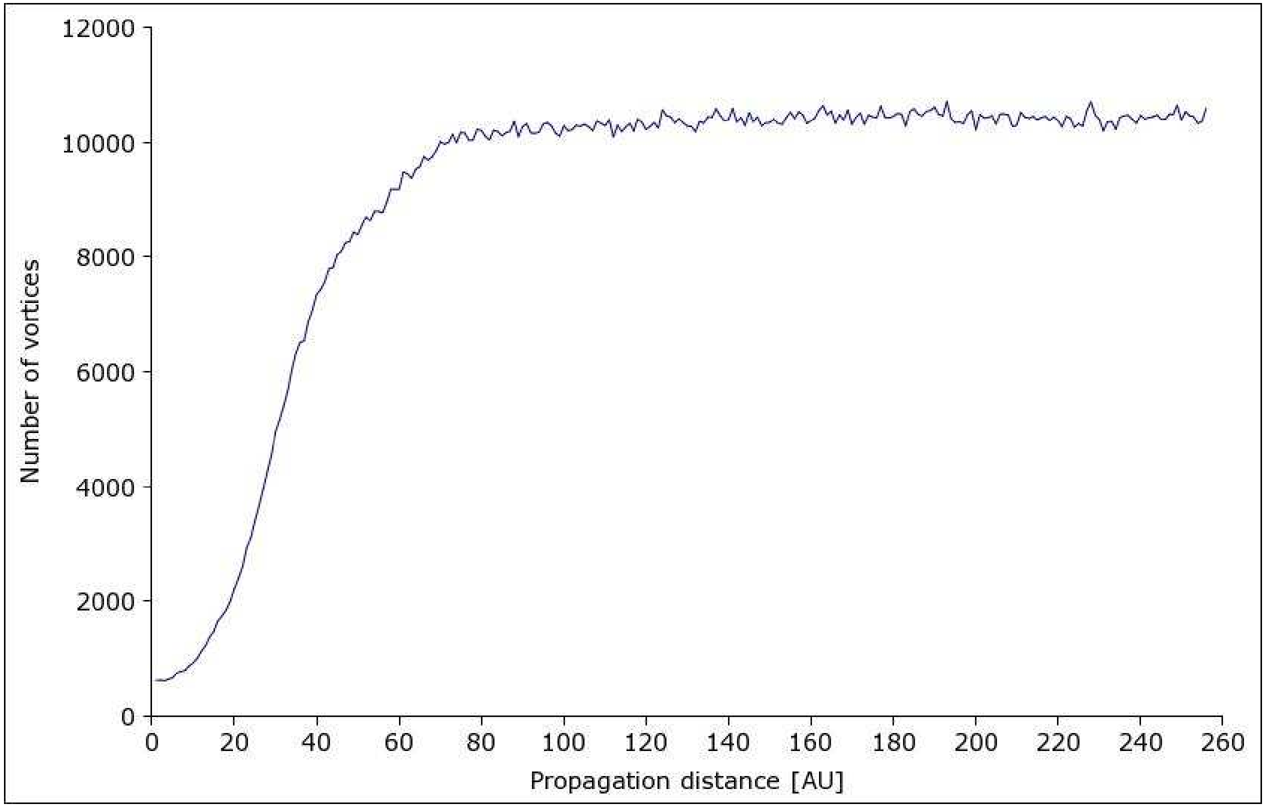
The maximum topological charge n in an area with circumference L:

$$\frac{n}{L} < \frac{1}{\lambda}$$

- Each vortex adds a phase cycle to circumference
- Area quadratic in radius, circumference linear in radius.
- Area increases $\Rightarrow$ spatial frequency on edge increases.
- If spatial frequency $>$ wavenumber $\Rightarrow$ only evanescent waves $\Rightarrow$ light depleted from exterior region.

Scale dependence of vortex fields

Number of vortices as a function of the propagation distance. The number increase up to some restoration scale.



Conclusions

- Shack-Hartmann wavefront sensor can be used to detect optical vortices.
- Forced annihilation of vortex dipole is achieved with the aid of a special phase function.
- Random wave fields contain random vortex distributions that are globally neutral.
- Artificial vortex fields containing vortices with the same topological charges in specific areas, are restored to equilibrium over some restoration scale.

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