An Efficient Approach to Node Localisation and Tracking in Wireless Sensor Networks

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Abstract—Localisation is one of the most important applications for wireless sensor networks since the locations of the sensor nodes are critical to both network operations and most application level tasks. Numerous techniques for localisation of sensor nodes that make use of the Received Signal Strength Indicator (RSSI) have been proposed because of the simplicity and low cost of implementation. However, most of the research thus far has regarded the RSSI technology as unsuitable for accurate localisation due to the limited accuracy inherent to the current ranging models. These models make the assumption that the antenna radiation pattern is omnidirectional in order to simplify the complexity of the algorithms. In this study, an accurate and efficient localisation method that makes use of an improved RSSI distance estimation model by including the antenna radiation pattern as well as nodes orientations is presented. Mathematical models for distance estimation, cost function and gradient of cost function, that can be used in a distributed localisation algorithm, are developed. This study also introduces a sensor data fusion approach, combining accelerometer data, RSSI, antenna radiation pattern and node orientation to reduce the computation complexity during the tracking phase. The proposed algorithm is implemented in Matlab. Simulation results show that the proposed approach increases the accuracy of existing methods using RSSI by up to 59%.

Index Terms—Antenna Radiation Pattern, Node Localisation, Sensor Data Fusion, Gauss-Newton Optimisation Method.

I. INTRODUCTION

Determining the location of nodes is one key application of Wireless Sensor Networks (WSN), for both civil and military applications. A reliable positioning system is critical in many WSN applications, especially for mobile ad-hoc networks. During the last decade, most of the research efforts have focussed on the improvement of the localisation accuracy and complexity. Distributed approaches using complex filtering and correction algorithms such as Kalman filters, Non Belief Propagation as well as other radio mapping techniques have been explored. Among these mapping techniques, Multidimensional Scaling (MDS) and Curvilinear Component analysis (CCA) are considered to be the most accurate and efficient [1], [2]. Ranging methods play a primary role in these localisation systems. The Received Signal Strength Indicator (RSSI) based ranging technique have received the most attraction. Specially designed antennas, filtering algorithms and other techniques have been developed and proposed in order to increase the localisation accuracy [3]. More complex solutions including pattern recognition techniques have also been proposed [4]. However, none of these methods have achieved a respectable accuracy when using the RSSI ranging technique [5].

The most common sources of ranging errors using RSSI include reflections on nearby objects, radio frequency noise, and variable characteristics of the communication channel. The biggest source for errors in distance estimation and hence localisation error for most localisation algorithms based on RSSI or Radio Frequency (RF) connectivity is the assumption that the antenna radiation pattern is perfectly circular or spherical in shape. It is therefore assumed that the formula for RSSI attenuation over distance, as described by the Log-Normal Shadowing Model (LNSM), is directly applicable. Lymboropoulos et al. [6] provided a detailed characterisation of signal strength properties and link asymmetries for the CC2420 radio using a monopole antenna. They showed that the antenna orientation effects are the dominant factor of the signal strength sensitivity in 3-dimensional network deployments.

However, in the real world, the pattern of radio transmitted signal at the antenna is neither a circular nor a spherical shape, and the path loss model is not valid due to problems caused by the sensor mote and the environment of the sensor field. A number of sensor systems are now deploying directional antennas due to their advantages such as energy conservation and better bandwidth utilisation.

Positioning systems are migrating towards hybridisation where data coming from heterogeneous technologies are fused to improve localisation accuracy and coverage. With the advances in Microelectromechanical systems (MEMS) technology, sensors such accelerometers and gyroscopes are found in many devices and are easy and cheap to implement in new designs.

In the algorithm proposed, orientation data, accelerometer data as well as the RSSI data, coupled with the antenna radiation patterns are proposed to provide an initial estimate from which the final position can be refined in few steps using an optimisation method such as the Gauss-Newton method.
A. Problem Statement

The problem to be solved by this study may be considered as follows. Given $N$ wireless sensor network nodes deployed randomly within a plane with some of them being mobile and given the positions of a small number $M$ of them that are considered as Anchor Nodes (ANs) together with the orientations of all the nodes and additional accelerometer measurements, a localisation algorithm that improves the accuracy of distance estimation using RSSI and efficiently finds the accurate positions of all nodes at fixed intervals in time needs to be developed.

II. RELATED WORK

While much attention has been paid to localisation accuracy and computational effort, the impact of irregular antenna radiation patterns have been often recognised with empirical studies having been conducted [6], [7]. However, its inclusion in the localisation algorithms considered were generally dismissed for future study.

Some authors have exploited the directivity of the radiation pattern as an approach for node localisation. In [8], the described approach requires a modification of the antenna and thereby its radiation pattern to ensure that the relative angle, $\varphi_{rel}$, obtained by pattern matching is unique and to maximise the accuracy. One possible choice is a beam pattern with one dominating lobe as used in [9], where the maximum of radiation pattern is used to determine the bearing. Due to the high noise of RSSI and a finite sampling angle interval, this approach may not be optimal in terms of accuracy and robustness. Another drawback is the significant reduction of the communication range in the direction outside the main lobe.

Attempts to model the radiation pattern described as Radio Irregularity Model (RIM) has been presented in [10]. However, none of these studies have developed a mathematical model or algorithm that includes the antenna radiation pattern and node orientation in localising the sensor nodes and no further research to include it in a localisation algorithm has been conducted.

III. MATHEMATICAL MODELLING

The first step for determining the location of a node is to find the distances between the respective nodes, which are assumed to be mobile, and some other nodes, which are assumed to be stationary. This is the so-called ranging phase.

A. Error Introduced By Radiation Pattern

From Fig. 1, the error introduced by the antenna radiation patterns can be expressed as shown in equation (1).

$$RSSI_{ij(Measure)} = RSSI_{ij(Actual)} + Ant_{Err}$$ (1)

B. Log-Distance Path Loss or Log-Normal Shadowing Model

The formula of the RSSI commonly used ranging method is the Log-Normal Shadowing Model formula given in equation (3).

$$RSSI_{ij} = P_{Tx}(j) - P_{L_{d0}}(j) - 10\eta \log \left( \frac{d_{ij}}{d_0} \right) + X(\sigma)$$ (3)

where:
- $P_{Tx}(j)$ is the power of the transmitted signal (dBm);
- $P_{L_{d0}}(j)$ the path loss at the reference distance $d_0$ (dB);
- $d_0$ is the reference distance (usually 1 m);
- $d_{ij}$ is the distance between node $i$ and node $j$ (m);
- $\eta$ is the path loss exponent also known as propagation index (unit-less);
- $X(\sigma)$ is a normal (or Gaussian) random variable with zero mean, reflecting the attenuation (dB) caused by flat fading.

This model does not take into account the effects of the antenna radiation patterns and nodes’ orientations. This study introduces these effect to obtain an improved RSSI model as shown in equation (4).

C. Improved RSSI Ranging Model

The improved ranging model is obtained by including the error introduced by the antenna radiation patterns to the value of RSSI received by the node. Equation (4) describes the improved model.

$$RSSI_{ij} = P_{Tx}(j) - P_{L_{d0}}(j) - 10\eta \log \left( \frac{d_{ij}}{d_0} \right) + Ant_{Err}$$ (4)
Since the biggest source of error is the one caused by antenna radiation pattern [6], \( X(\sigma) \) found in equation (3) is assumed to be negligible compared to \( \text{AntErr} \) in this study as shown in equation (5), therefore it does not appear in equation (4).

\[
X(\sigma) \approx 0
\]

(D) Optimisation Problem Formulation

The aim of the localisation approach is to match the computed distances based on the node’s estimated positions and the measured ones using the RSSI ranging technique.

(E) Cost function

For the \( M \) pairs of nodes that are in range of each other:

\[
f = \sum_{k=1}^{M} \| d_{ij}(k) - \tilde{d}_{ij}(k) \| \tag{6}
\]

where:

\[
d_{ij}(k) = \sqrt{(x_i(k) - x_j(k))^2 + (y_i(k) - y_j(k))^2} \tag{7}
\]

and

\[
\tilde{d}_{ij}(k) = 10 \log \left( \frac{P_{R_t}(k) - P_{E_{\text{at}}}(k) - \text{RSSI}_{ij}(k) + \text{AntErr}(k)}{10^\eta} \right) \tag{8}
\]

(F) Constraints

The equality constraints are imposed by the values of the positions of the anchor nodes as well as localised nodes. Let \( k \) represent the ID of the anchor nodes or localised nodes. Therefore,

\[
A_{eq}X = b_{eq} \tag{9}
\]

with \( A_{eq}(n, k) = 1 \) and \( b = [x_k, y_k] \) for

- \( k = 1 : P \), \( P \) being the number of anchors or reference points in the cluster;
- \( n \in \{ \text{anchor IDs} \} \) and \( \text{length}(\text{anchor IDs}) = M \).

(G) Optimisation Method

A gradient based optimisation method is used to calculate the position of the nodes. Newton's optimisation method uses the gradient and the Hessian matrix of second derivatives of the function to be minimised. Unlike Newton’s method, the Gauss-Newton method has the advantage that second derivatives, which can be challenging to compute, are not required. However, it can only be used to minimise a sum of squared function values, which is the case for the localisation problem formulation this study attempts to solve. The Gauss-Newton algorithm iteratively finds the minimum of the sum of squares. The update mechanism is described in equation (10).

\[
X_{k+1} = X_k - (J_k^T J_k)^{-1} J_k^T \xi_k \tag{10}
\]

where \( J_k \) is the Jacobian matrix(gradient) of the cost function estimated at \( X = k \) and \( \xi_k \) is the residual error matrix form by the individual terms of the cost function computed as will be shown in equation (17).

(H) Distributed and Cooperative Localisation Based on Anchor Node Position Propagation

In this study, a distributed approach is proposed in which each cluster head node estimates locally the positions of the cluster members based on range measurements obtained from neighbours and some prior information, such as positions of other anchor nodes in the cluster, if available.

IV. ALGORITHM DESIGN AND IMPLEMENTATION

To ensure that the proposed algorithm takes the antenna radiation pattern into account, the orientations of the antenna of each sensor node must be known. An earth magnetic sensor is used for this effect. To further improve the accuracy and computational speed, accelerometer data are fused RSSI and antenna orientation measurements. To achieve these objectives, this study uses the LSM303DLHSS sensor from ST Microelectronics. This MEMS sensor chip integrates a tri-axial magnetic sensor and a tri-axial accelerometer.

The node positions are arranged in a matrix \( X \) in 2-D plan, with indices corresponding to the indices of the corresponding node in the cluster vector \( X \). The set of measured distances between node \( i \) and the neighbouring node \( j \) are given by \( d_{ij} \) and the Euclidean distance between the same nodes is given by \( \tilde{d}_{ij} \). The algorithm for localising the neighbouring nodes at the distributed (local) level from only the anchor node position is derived from graph theory and is designed using the following steps:

A. Step 1: Pre-computing

Create a cluster of nodes in its immediate neighbourhood (N nodes). Only M pairs can communicate between them. For each pair \( \{ s(k), r(k) \} \), where \( s(k) \) is the \( k \)th sending node and \( r(k) \) is the \( k \)th receiving node, only the directed connection between two nodes starting from \( s(k) \) and ending at \( r(k) \) are considered. From the connectivity graph, with \( k = 1 : M \), the connectivity matrix also known as incidence matrix can be calculated

\[
1) \text{Connectivity Matrix: } C = \begin{bmatrix}
C_{11} & C_{12} & \ldots & C_{1k} & \ldots & C_{1N} \\
C_{21} & C_{22} & \ldots & C_{2k} & \ldots & C_{2N} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_{11} & C_{12} & \ldots & C_{ik} & \ldots & C_{iN} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_{M1} & C_{M2} & \ldots & C_{Mk} & \ldots & C_{MN}
\end{bmatrix}
\]

where

\[
C_{ik} = \begin{cases}
+1 & \text{if } i = s(k) \\
-1 & \text{if } i = r(k) \\
0 & \text{otherwise}
\end{cases}
\tag{11}
\]

\[
2) \text{Radiation patterns of nodes } j \text{ in (dB): } \text{Patrn}_j(k,:)=10 \log (\text{RadPattern}(r(k)))
\tag{12}
\]
B. Step II: Computation of the Cost Function

To calculate the cost function as defined in equation (6), the following terms must be computed as shown in the following steps, where \( k = 1 : M \).

1) **Euclidean distances**:

Let \( \delta_{xy} \) be the matrix containing the differences \([(x_i - x_j), (y_i - y_j)]\), where \( i = s(k), j = r(k) \) and \( k = 1 : N \)

\[
\delta_{xy} = \begin{bmatrix} x(i(1)) - x(j(1)) & y(i(1)) - y(j(1)) \\ x(i(k)) - x(j(k)) & y(i(k)) - y(j(k)) \\ \vdots & \vdots \\ x(i(M)) - x(j(M)) & y(i(M)) - y(j(M)) \end{bmatrix}
\]  

(13)

\( \delta_{xy} \) can be obtained in as shown in equation (14).

\[
\delta_{xy} = C * X
\]  

(14)

2) **Vector of euclidean distances**:

\[
d_{ij} = \begin{bmatrix} d_{ij}(1) \\ d_{ij}(k) \\ \vdots \\ d_{ij}(M) \end{bmatrix} = \sqrt{\text{sum}(<\delta_{xy}, \delta_{xy}>, 2)}
\]  

(15)

3) **Vector of Estimated distances**:

\[
\tilde{d}_{ij}(k) = 10.\text{Exp}(k)
\]  

(16)

where

\[
\text{Exp}(k) = \frac{(P_{T_{xx}} - P_{L_{do}} - \text{RSSI}_{measured} + \text{AntErr}(k))}{10\eta}
\]

4) **Sign vector of the cost function**:

The cost function is made up of the sum of residual errors expressed in equation (17).

\[
\xi = \begin{bmatrix} d_{i,j} - \tilde{d}_{i,j} \\ d_{j,k} - \tilde{d}_{j,k} \\ \vdots \\ d_{M,j,M} - \tilde{d}_{M,j,M} \end{bmatrix}
\]  

(17)

The terms of the vector \( \xi \) are not always all positive. Therefore the absolute values of the terms of vector \( \xi \) must be considered when calculating the cost function. Consider a vector \( S_\xi \) as being the vector sign of \( \xi \). The absolute value \( \| \xi \| \) would be obtained as a dot product multiplication of the vectors \( \xi \) and \( S_\xi \). Therefore,

\[
S_\xi = \xi / \text{abs}(\xi)
\]  

(18)

where \( \cdot / \cdot \) represents a dot division sign.

5) **Cost function**:

The cost function is calculated by summing up the absolute values of the residual errors computed in equation (17). Therefore,

\[
f = \sum_{k=1}^{M} \| \xi(k) \|
\]  

(19)

C. Step III: Computation of the Gradient of the Cost function

The gradient of the function is required when using Newton and quasi-Newton optimisation methods. The vector \( S_\xi \) computed in equation (18) is used to correctly affect the sign of terms during the computation of the gradient of the cost function.

\[
\nabla f = S_\xi \ast (\nabla d_{ij} - \nabla \tilde{d}_{ij})
\]  

(20)

The computation of equation (20) is divided into the following steps:

1) **Computation of** \( \nabla d_{ij} \):

\[
\nabla d_{ij} = C^T * \left( \frac{x_{ij} - x_{ij}}{d_{ij}(1)} \right)
\]

(21)

2) **Computation of** \( \nabla \tilde{d}_{ij} \):

For the pairs \( k = 1 : M \) of nodes, we have:

\[
\nabla \tilde{d}_{ij} = \frac{\partial}{\partial x_i} \left( 10 \cdot \text{Exp}(k) \right)
\]

(23)

and

\[
\nabla \tilde{d}_{ij} = \frac{\partial}{\partial y_i} \left( 10 \cdot \text{Exp}(k) \right)
\]

(24)

Where \( i = 1 : N \). By developing the equations(23) and (24), we obtain:

\[
\nabla \tilde{d}_{ij} = \frac{\log_e(10)}{10\eta} \ast C^T \ast (\nabla_{ij}(\text{AntErr}(x, y)))
\]  

(25)

where :

\[
\nabla\text{AntErr} = (\nabla\text{Ant}(\phi_j)) - \nabla\text{Ant}(\phi_i))
\]

The gradient of the cost function is obtained by substituting the Equations (25) and (22) in equation (20).

D. Initial Position

Locating initial positions in the convex hull to not only reduces the number of iterations of the process of localising the nodes, it also avoids flip ambiguities of the localised nodes. The efficient computation of the convex hull requires a Minimum Spanning Tree (MST) as a starting point. There are simple, elegant, fast algorithms to find them. In this study, the Dijkstra-Jarvik-Prim algorithm for the MST and the Graham’s Scan Algorithm for the Convex Hull are used as illustrated in Fig. 2.
There is another method of integrating that uses the Simpsons rule. Unlike the other methods, this approach requires a future sample of the integrand, X, to get the current sample of the integrated signal, Y, so it cannot be performed in real time. For all these reasons explored above, this study used the trapezoidal integration method.

V. Simulation Results

The Gauss-Markov mobility model was used to simulate a realistic mobile network and to generate accelerometer data. Corresponding data (RSSI, Acceleration) were also updated accordingly. The Gauss-Markov model was implemented using the following Equations (28) and (29).

\[ v_n = \alpha v_{n-1} + (1 - \alpha) \dot{v} + \sqrt{(1 - \alpha^2)} \sigma_n \] 
\[ \theta_n = \alpha \theta_{n-1} + (1 - \alpha) \dot{\theta} + \sqrt{(1 - \alpha^2)} \sigma_n \]

where \( v_n \) and \( \theta_n \) represent the velocity (speed) and direction (heading) at stage n of the model, \( \dot{v} \) and \( \dot{\theta} \) represent the average speed and heading respectively and \( \sigma_n \), a random number from a Gaussian distribution with zero mean and variance \( \alpha \). \( \alpha \) is a parameter used to fine tune the Gauss-Markov mobility model.

The radiation pattern can be found in the data sheet of the sensor node’s radio module or can be measured, as explained in the experiment below.

Only 36 measurements were taken and considered in the determination of the radiation pattern. A coarse radiation pattern was obtained from the measurements. After interpolation, a more refined radiation pattern was obtained with a resolution of 1° as depicted in Fig. 3.

Fig. 4 shows the localisation result of the algorithm applied to 50 nodes randomly deployed. Fig. 5 outlines the accuracy of the developed algorithm by comparing its localisation errors to the approach that does not make use of the antenna radiation pattern, both using the Gauss Newton optimisation method.

The algorithm was tested for fidelity with a performance metric found in literature.

The precision of the localisation methods consisted of repeating a number of times the localisation process for randomly deployed nodes and comparing the average localisation

\[ Y(n) = \frac{1}{f_s} \sum_{k=0}^{n} X(n-k) = Y(n-1) + \frac{1}{f_s} X(n) \] 
\[ Y(n) = Y(n-1) + \frac{1}{2f_s} [X(n-1) + X(n)], n > 0 \]

F. Numerical Integration Methods

The simplest way to perform numerical integration is to use the rectangular integration method. This method uses an accumulator to sum all past sampled inputs and the current input sample and divide it by the sampling rate. Rectangular integration is represented by the difference equation expressed in equation (26).

\[ Y(n) = \frac{1}{f_s} \sum_{k=0}^{n} X(n-k) = Y(n-1) + \frac{1}{f_s} X(n) \] 

where \( x \) is the integrand, \( Y \) is the output of the integrator, and \( f_s \) is the sampling frequency. Another numerical integration method uses the trapezoidal rule. The results are more accurate with this method than with the rectangular method. The difference equation for trapezoidal integration is shown in equation (27).

\[ Y(n) = Y(n-1) + \frac{1}{2f_s} [X(n-1) + X(n)], n > 0 \]
The algorithm developed in this study was shown to be accurate as shown in the results presented in Fig. 6. The scalability of the proposed approach was demonstrated by considering a thousand nodes randomly deployed and the result can be seen in the results shown in Fig. 7.

VI. CONCLUSION AND FUTURE WORK

This paper has considered the development of a framework to include a mathematical model of the antenna radiation pattern in a WSN node localisation algorithm. The paper has demonstrated a successful implementation of the algorithm using a Gauss-Newton optimisation method. Simulations results show that the model improves the accuracy of localisation by 59%. The model was developed for a 2-D plan. It is proposed that future studies consider the implementation of the developed model in other localisation methods and that it be extended to 3-D model.

REFERENCES