Vitality of optical vortices

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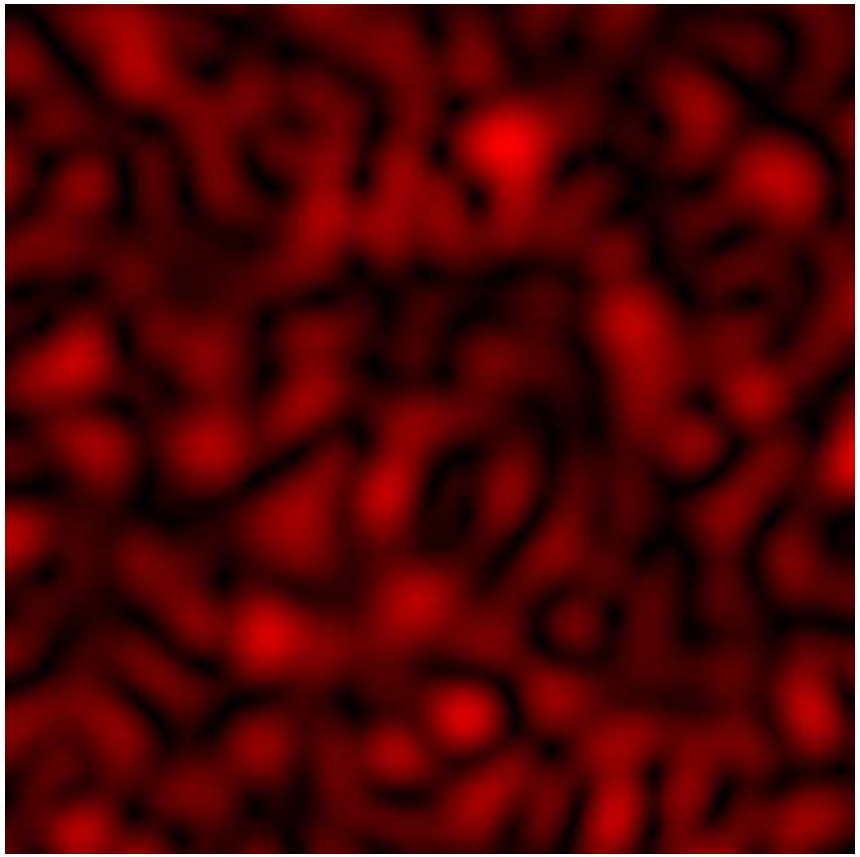
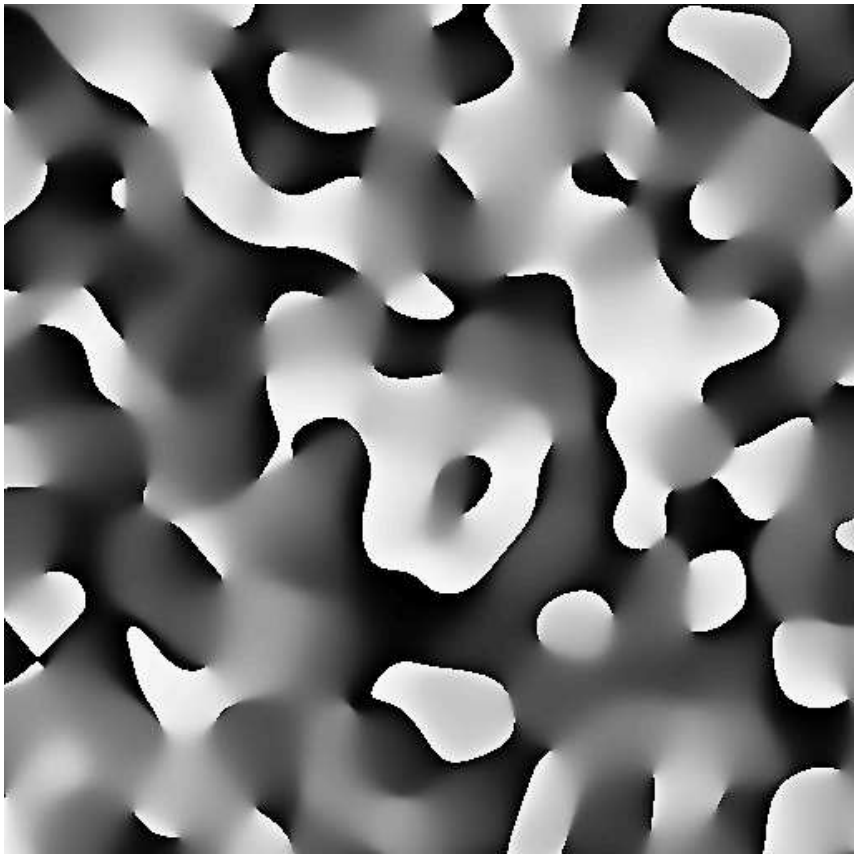
Speckle



Amplitude



Phase

Vortex conservation

Topological conservation: the net flow of topological charge into a finite region of space is zero

Conservation equations:[a]

$$\partial_z n_p + \nabla \cdot \mathbf{J}_p = \mathcal{C} - \mathcal{A}$$

$$\partial_z n_n + \nabla \cdot \mathbf{J}_n = \mathcal{C} - \mathcal{A}$$

n_p (n_n) — positive (negative) vortex density
$\mathbf{J}_p$ ($\mathbf{J}_n$) — current for the positive (negative) vortices
$\mathcal{C}$ ($\mathcal{A}$) — creation (annihilation) events per unit volume

[a] FS Roux, Opt. Commun. 283, 4855-4858 (2010)

Conservation of V and T

For $V = n_p + n_n$ and $T = n_p - n_n$:[a]

$$\partial_z V + \nabla \cdot \mathbf{J}_V = 2(\mathcal{C} - \mathcal{A})$$

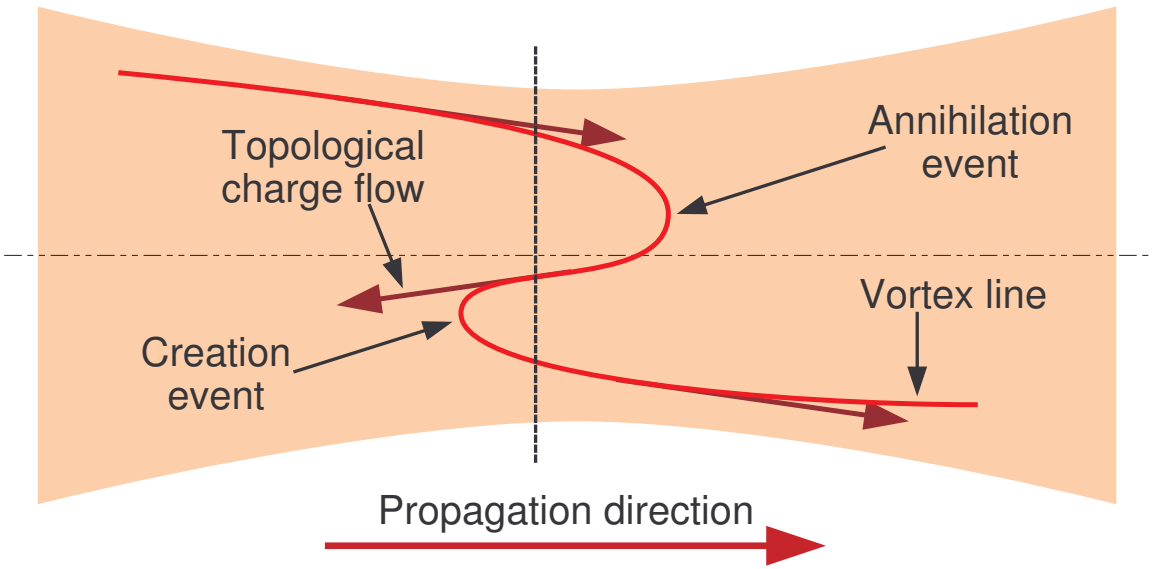
$$\partial_z T + \nabla \cdot \mathbf{J}_T = 0$$

V — vortex density
T — topological charge density
$\mathbf{J}_V$ — current for the vortex density
$\mathbf{J}_T$ — current for the topological charge density

[a] FS Roux, Opt. Commun. 283, 4855-4858 (2010)

Vortex line and critical points

Vortex location: $\mathcal{R}e\{g(\mathbf{x})\} = \mathcal{I}m\{g(\mathbf{x})\} = 0$



Topological charge flow $\rightarrow$ vorticity:[a] $\Omega = \frac{\mathrm{i}}{2}\nabla g(\mathbf{x}) \times \nabla g^*(\mathbf{x})$

Sign of z component of vorticity indicate topological charge

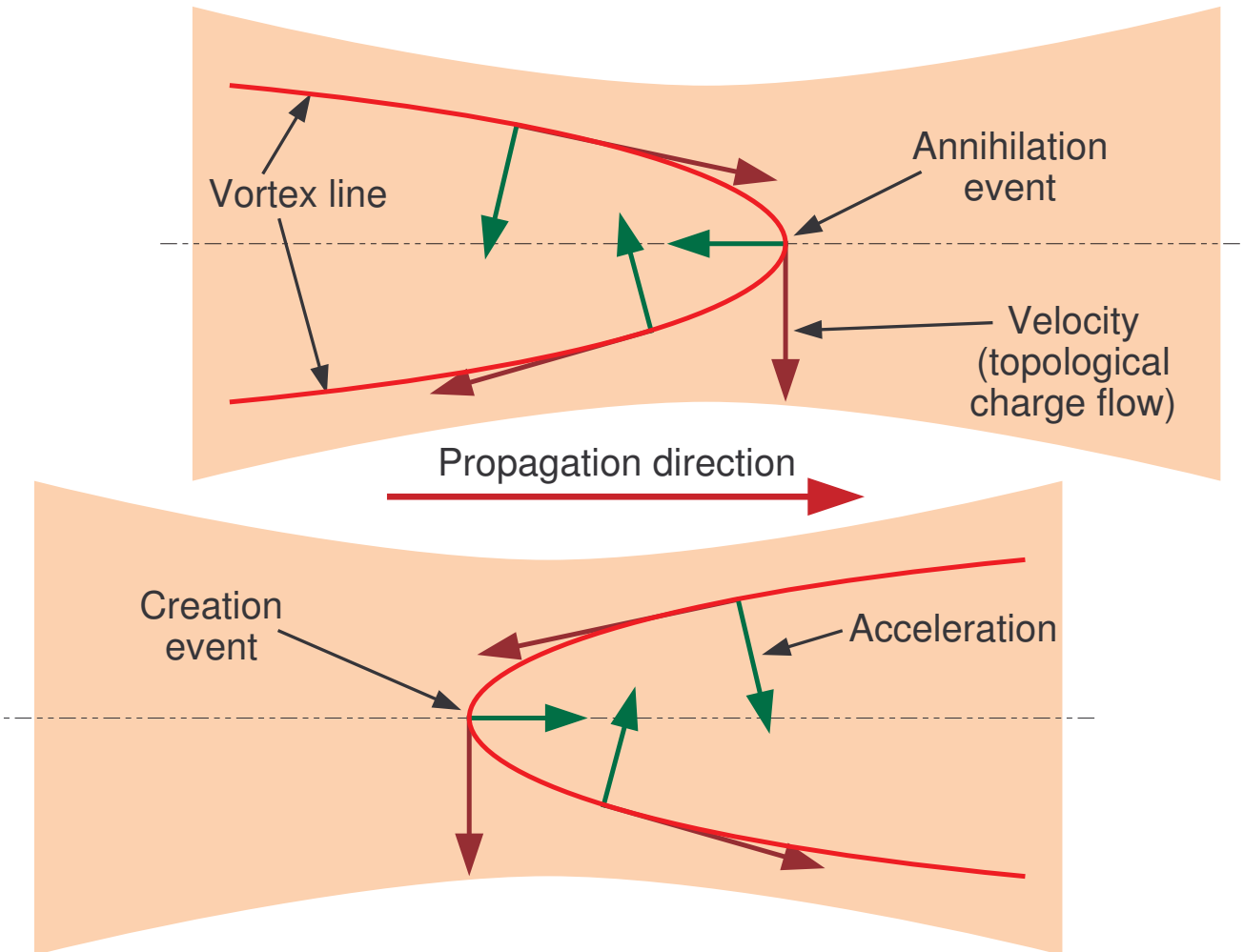
$$\Omega \cdot \hat{z} = \frac{\mathrm{i}}{2}(g_x g_y^* - g_y g_x^*) = 0,$$

[a] MV Berry and MR Dennis, Proc. R. Soc. Lond. A 456, 2059-2079 (2000)

Acceleration vector

Velocity vector: $\mathbf{v} = \dfrac{\Omega}{|\Omega|}$

Acceleration vector: $\mathbf{a}(t) = \partial_t \mathbf{v}(t)$



Vitality

The vitality at a critical point:[a]

$$\mathcal{V} = H_1 H_3 + H_2 H_4$$

with

$$H_1 = g^*_{xx} g_y + g_{xx} g^*_y + g^*_{yy} g_y + g_{yy} g^*_y$$

$$H_2 = g^*_{xx} g_x + g_{xx} g^*_x + g^*_{yy} g_x + g_{yy} g^*_x$$

$$H_3 = \mathrm{i}\left(g^*_{xx} g_y - g_{xx} g^*_y - g^*_{xy} g_x + g_{xy} g^*_x\right)$$

$$H_4 = \mathrm{i}\left(g^*_{yy} g_x - g_{yy} g^*_x - g^*_{xy} g_y + g_{xy} g^*_y\right)$$

The sign of $\mathcal{V}$ indicates whether the critical point is an annihilation event or a creation event.

[a] FS Roux, Opt. Lett. 38, 3895-3898 (2013)

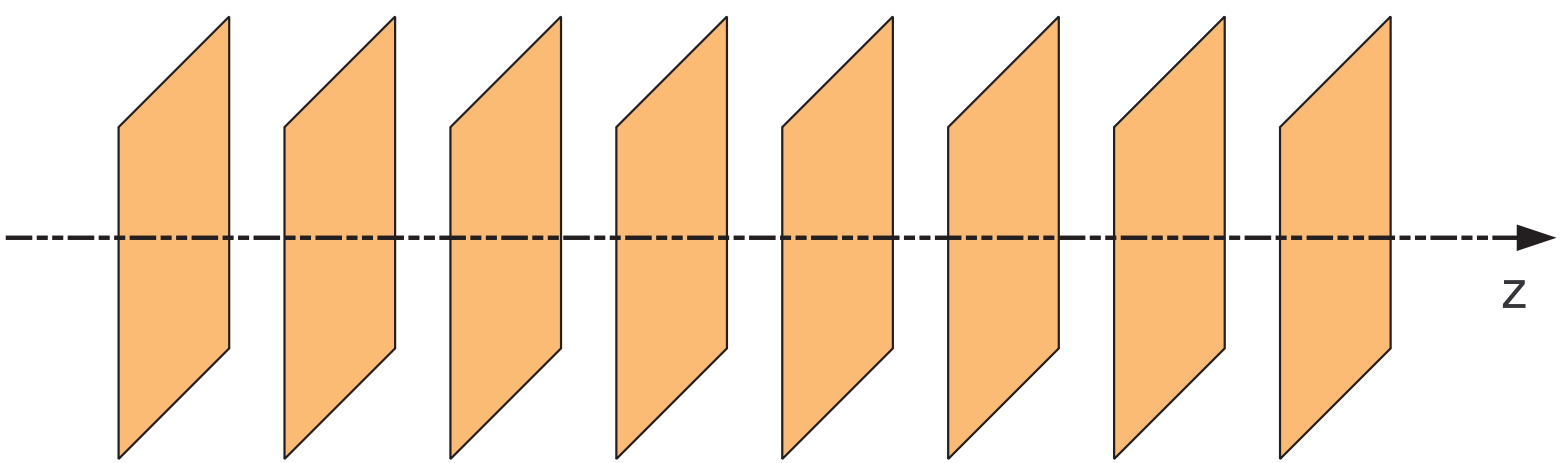
Numerical simulation

Speckle field: $\psi(\mathbf{x}) = \sum_n \chi_n \exp(-i\mathbf{k}_n \cdot \mathbf{x})$

χ_n — random complex coefficients
$\mathbf{k}_n$ — random propagation vectors

($\mathbf{k}_n$ restricted to small cone angle around z-axis.)

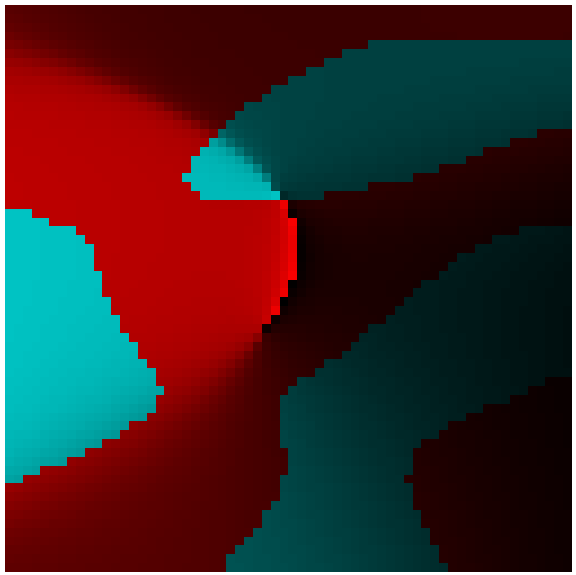
Reconstruct at series of z values
$\rightarrow$ sequence of 2D optical fields.



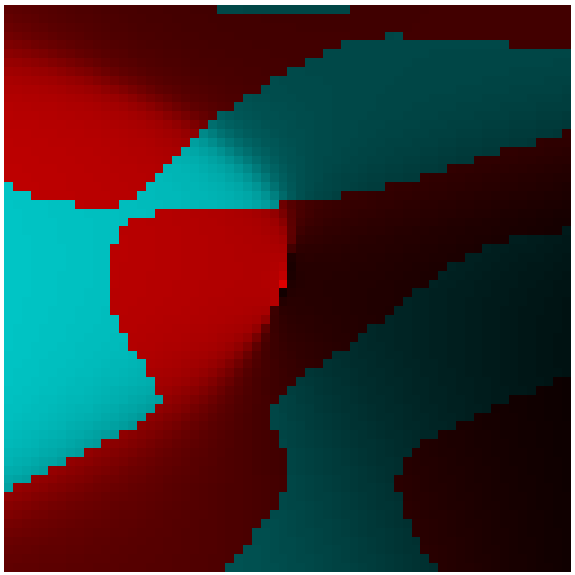
Annihilation example

Sign of the vitality:
cyan = positive vitality
red = negative vitality

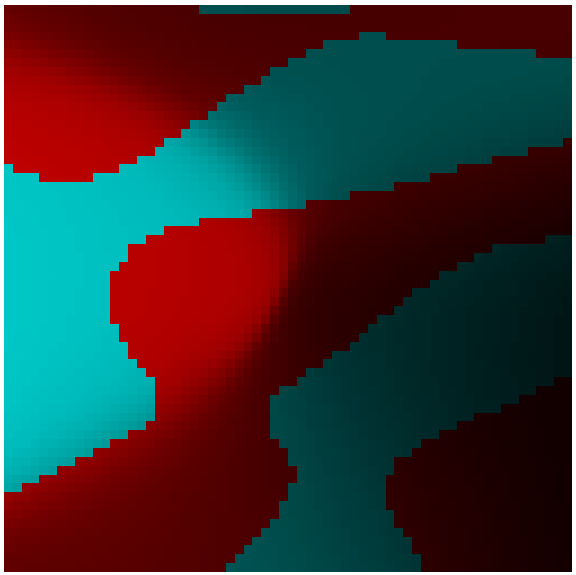
(a)



(b)



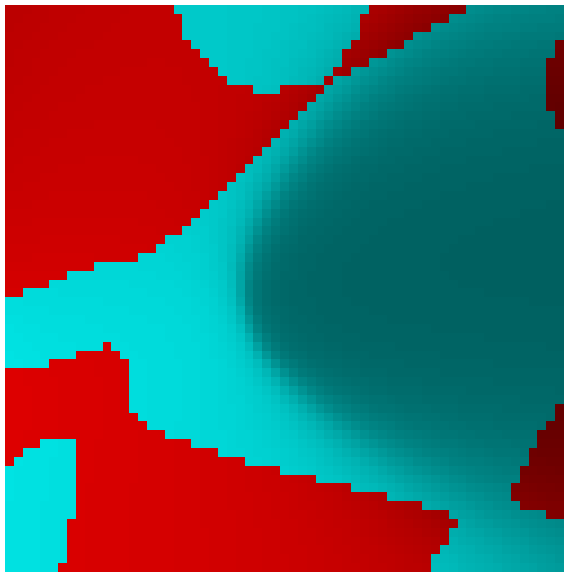
(c)



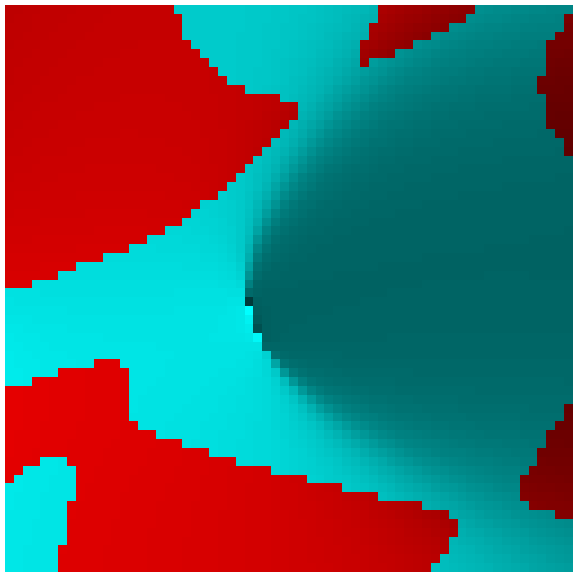
Creation example

Sign of the vitality:
cyan = positive vitality
red = negative vitality

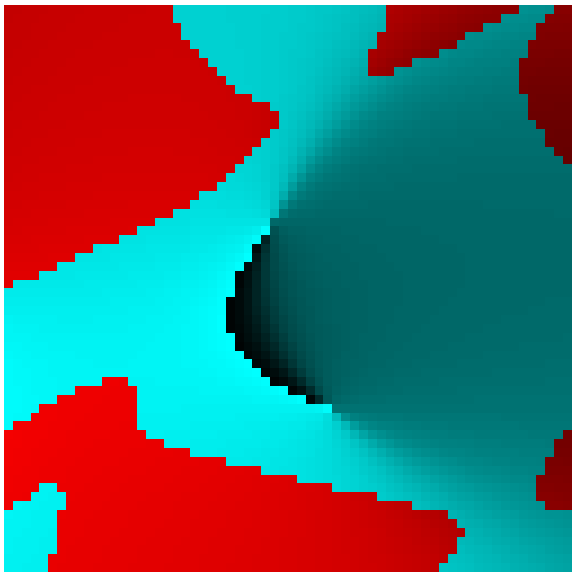
(a)



(b)



(c)



Summary

- Derived a quantity (the vitality) to distinguish between vortex dipole creation and annihilation events.
- Vitality is expressed in terms of the transverse 1st and 2nd order derivatives of the optical field.
- It can be used to compute the probability density for the difference of creation and annihilation events.
- Only gives unambiguous identification of the type of event at the location of a critical point.