Topology of the landscape of optimally controlled transitions in a multilevel system

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INTRODUCTION

A model for selectively exciting a specific quantum level of a multilevel system was developed at the CSIR National Laser Centre[1]. This model utilises adaptive feedback control and beam shaping to optimise the population within a specific vibrational level in the system. This study will concentrate on studying the structure of the control landscape of this particular problem.

According to a theoretical analysis[2], the control landscape of many quantum control problems has a very favourable topology regardless of the detailed nature of the Hamiltonian, provided that one has full control of the system. It is obvious that full control is not possible in a practical experiment and this study will investigate the influence of experimental and other limitations on the control landscape. Depending on the outcome of the investigation, the applicability of various optimisation techniques will be investigated. In particular, gradient-based optimisation techniques will be investigated and their results will be compared with the results obtained by the more traditional (in the sense of quantum control schemes) genetic type optimisation techniques.

ILLUSTRATION OF METHODS

$$f(x,y) = x\exp\left(-0.0125x^2 - y^2\right)\left(\sin\left(\sqrt{x^2+y^2}\right)\bullet\cos\left(\sqrt{x^2+y^2}\right)\right)$$

$$f(x^*,y^*) = -1.892886607 \qquad [x^*\ y^*] = [-7.06\ 0] \qquad (1)$$

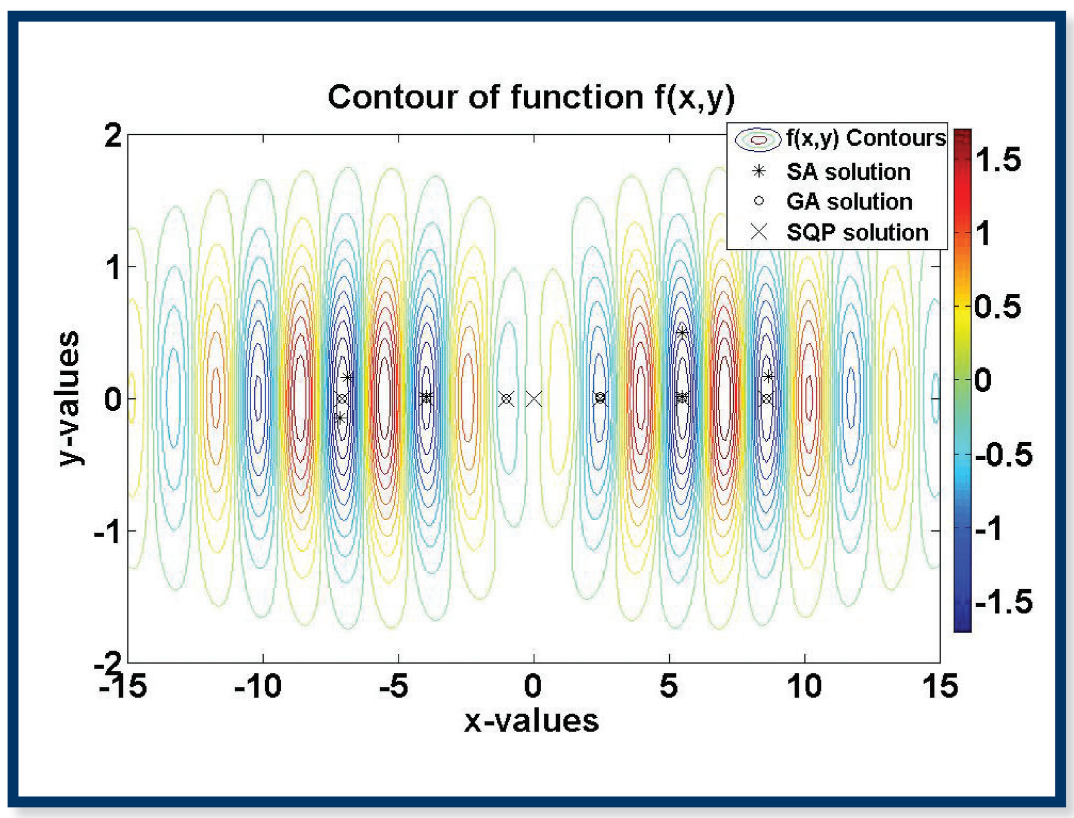

Figure 1: Contour plot of equation 1 with optimum point for GA, SA and gradient-based method

THE MODEL

Figure 2 (right): An adaptive feedback control (AFC) experiment is a closed loop experiment in which a learning loop shape pulses with a spatial light modulator (SLM), or any other pulse shaping devise, to obtain a certain outcome. The learning loop algorithm requires some initial guess of pulse. The pulse that is formed with the shaping devices interacts with the system and measurements are taken and fed back to the algorithm. The process stops when predefined conditions are met.

This model utilises adaptive feedback control and beam shaping to optimise the population within a specific vibrational level in the system.

RESULTS

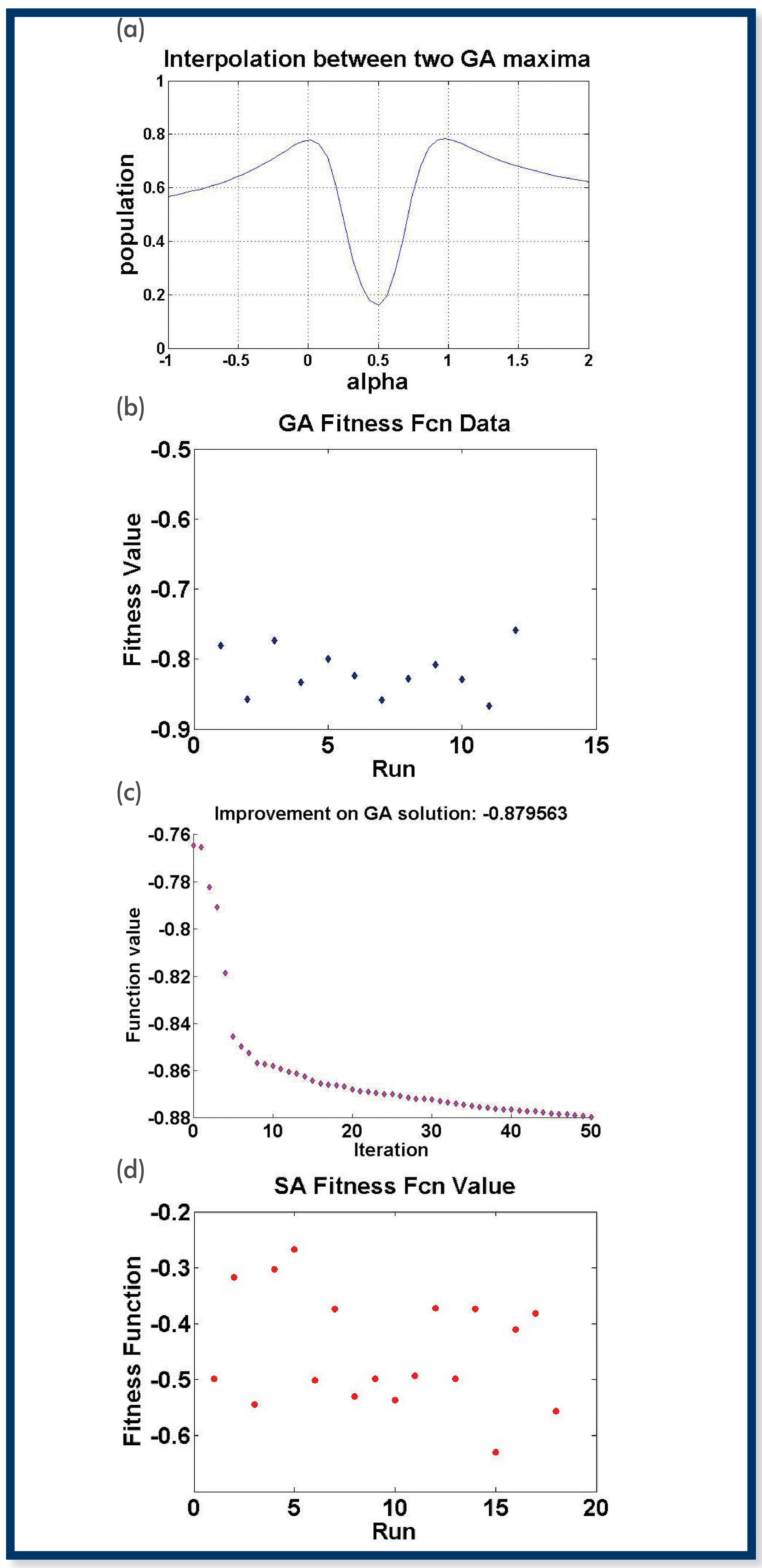

Figure 2: (a) Interpolation between GA maxima, (b) GA fitness function at each run, (c) An improvement of a GA solution using the gradient-based method, and (d) SA function values for different runs

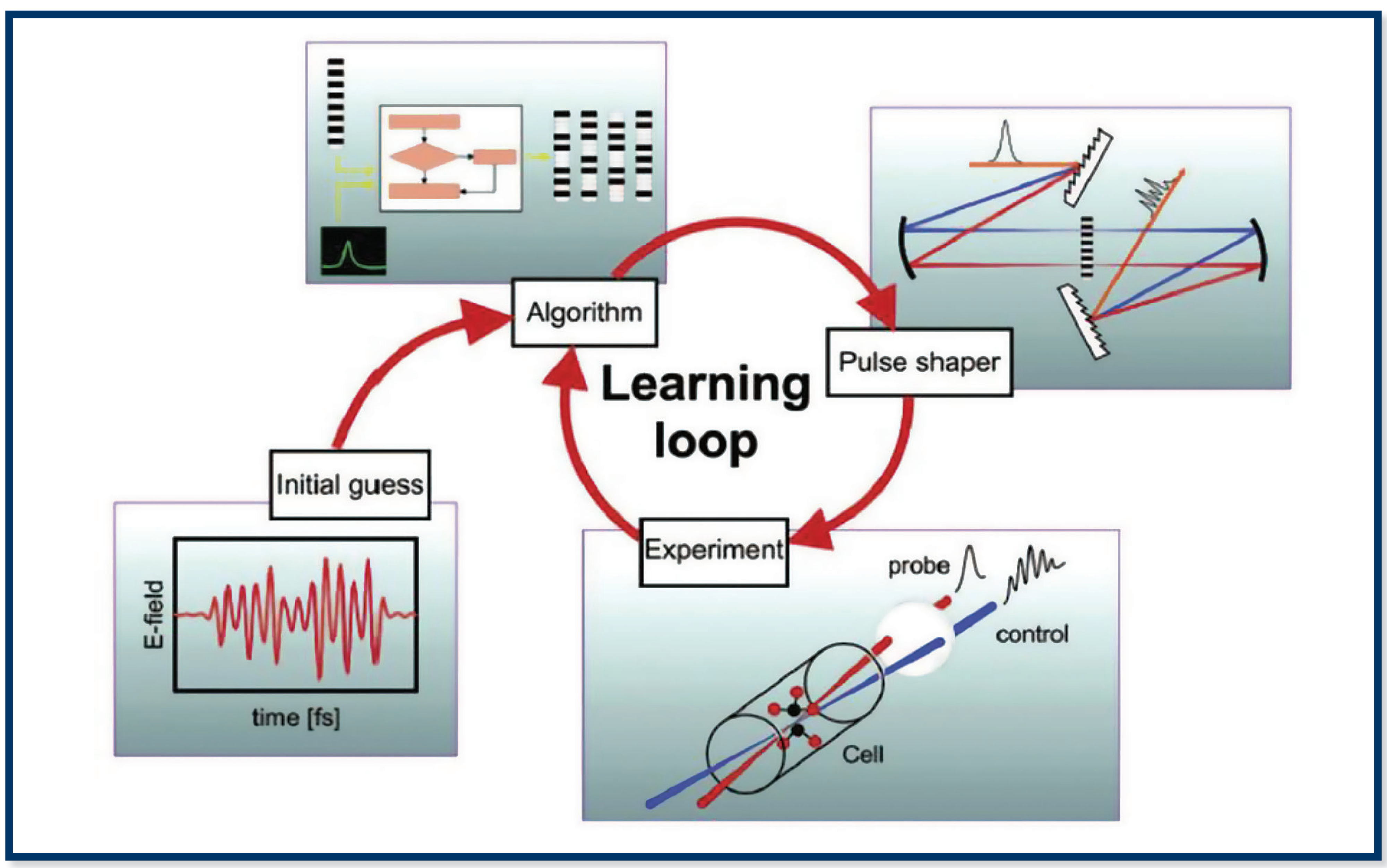

Table 1: Difference between GA solutions $\|x-y\| = \sqrt{\sum_{i=0}^{n}|x_i - y_i|^2}$

Fitness value (%)	78.132	85.733	77.517	83.3384
Norm ($x_i - x_j$)	X_1	X_2	X_3	X_4
X_1	0			
X_2	8.329	0		
X_3	9.0854	9.9078	0	
X_4	10.044	10.1247	8.843	0
X_5	9.2657	9.766	9.3524	10.7809

Table 2: Improved GA solution using a gradient-based method and the difference between their points $\|x-y\| = \sqrt{\sum_{i=0}^{n}|x_i - y_i|^2}$

Genetic algorithm (GA) function value (%)	Gradient method function value (%)	Distance between GA and gradient method solution points
83.383	84.7823	14.9344
86.729	86.8039	7.4327
77.517	87.9563	14.7516

CONCLUSION

We have shown that the simulated annealing (SA) method is not robust with regard to our model.

Although the genetic algorithm (GA) performs better, it exhibits trapping behaviour, since we can get an improvement in its solutions through gradient-based methods.

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