Influence of Errors in the Dimensions of a Switched Parasitic Array on Gain and Impedance Match

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Abstract- The dimensions of an antenna system defines its performance. This implies proper calibration and fabrication process are vital. Thus, there is a need to investigate the influence of the variations in the dimensions of the antenna on its performance. A switched parasitic array antenna is modelled in MATLAB. The variations in the antenna dimensions: length, thickness and spacing of the array elements, are randomly generated based on the Gaussian distribution. The overall influence of these variations on the antenna performance attributes (e.g. gain and impedance match) is estimated using a Monte Carlo simulation. The simulation results demonstrate that the combined effect of all variations in the structural parameters quantify the impact of these variations on the performance of the antenna system, for the given antenna specifications.

Index Terms—Monte Carlo simulation; Impedance; Gain; Antenna arrays; Error analysis; ESPAR.

I. INTRODUCTION

With an increasing demand for wireless broadband, especially for rural connectivity, it is important to consider network's components that are energy efficient yet offering great performance to deliver broadband services to the rural communities. Antennas are amongst the fundamental components of any communications system because they can directly influence the quality of service, the range and capacity rendered by the networks, as well as the cost of establishing and/or maintaining such networks [1].

Multiple element (array) antennas such as smart antennas and Multiple Input Multiple Output (MIMO) systems are considered to be the technologies to address many of the current issues in the wireless communication networks. However, the use of the aforementioned technologies introduces some design challenges, such as increased cost, power consumption, complexity and array size, some of which are unsuitable and/or unaffordable for certain applications. Arrays based on parasitic array technology have received much attention as the alternative technology to aid in addressing the energy efficiency and high performance antenna systems [2–7].

In designing the electronically controllable parasitic arrays (e.g. electronically steerable passive array radiator, ESPAR), the control and structural parameters have to be well optimized to achieve good performance [5]. As every system performs better under certain (optimized) specifications, any variations in the specifications can affect the system's performance. The variations in the system's specifications, e.g. mechanical dimensions, may be as a result of random errors. Although random errors may be minimal, they have influence on the antenna performance attributes such as gain, side lobes, and beam positioning precision [8–11] and require to be quantified.

Several studies in the literature have presented the effects of random errors on the antenna performance, for linear or planar arrays consisting of only the active elements [8], [9], [12], [13]. For instance, effects of the random errors in linear and planar arrays, with focus on the side-lobe level [13] directivity (or gain) [8], [9], [14], and beam pointing accuracy [14] have been presented. In addition, the effects of the mutual coupling error on the input admittance of antenna arrays have been studied in [13]. In [13], the effects of mutual coupling are undesirable, whereas in parasitic arrays mutual coupling is important for the general performance of the antenna.

A circular switched parasitic array (SPA) antenna gain and impedance sensitivity analysis, based on variations in each individual structural parameter are presented in [6], [15]. The assumption made in [15] is that, only one parameter is varying at a time and all other parameters are fixed as per the antenna specifications. However, there are less chances of such situation, especially when the antenna has more than three (length, thickness and spacing in between the elements) of the structural parameters. Therefore, an investigation on the combined effect of the variations in all structural parameters is desirable.

In this paper, a circular switched parasitic array (SPA) antenna operating at the 2.4 GHz is considered. The overall influence of the variations in the antenna's mechanical dimensions (structural parameters) is investigated based on a Monte Carlo simulation [16]. The method demonstrated that the combined effect of all variations (total error) has a significant effect on the performance of the SPA antenna.

The rest of the paper is organized as follows. Section II presents the antenna system under study. In section III, the proposed approach to investigate the effects of random errors on the performance attributes of the switched parasitic array (SPA) antenna is introduced. Section IV overviews a simulation method based on Monte Carlo simulation. In the section, the simulation results and analysis are presented. The study is concluded in section VI.

II. MODELLED SPA ANTENNA SYSTEM

The work considers a single ring circular switched parasitic array (SPA) antenna, consisting of five dipole elements (N=5) in free space, with one central active element surrounded by four parasitic elements (P = 4). All antenna elements are vertically oriented, and perpendicular to the ground plane (x-y axis), as illustrated with Fig. 1. The configuration with only one parasitic element open-circuited and other parasitic elements short-circuited is chosen to be used.

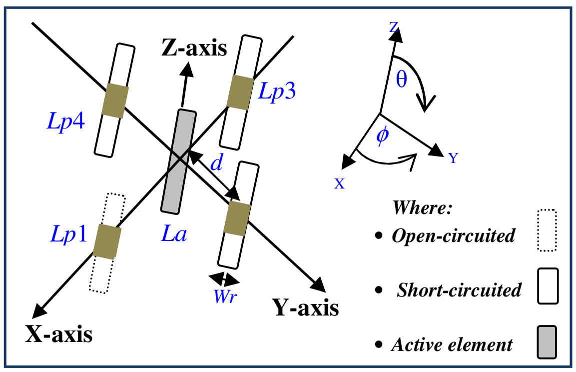

Fig. 1: A geometry of the SPA antenna with five elements: one central active element surrounded by 4 parasitic elements [6].

The specifications for the modelled SPA antenna are presented in Table 1. All parasitic elements are of equal length for the purposes of symmetry of the circular SPA antenna [6]. Also, for simplicity, all the elements (active and parasitic) have the same thickness. The antenna specifications are adopted from the proposed SPA antenna prototype in [6]. The antenna operates at the 2.4 GHz.

Table 1: Specifications of the circular SPA antenna

Parameter	*Value*
Length of the active element, L_a	52 mm
Length of all parasitic elements, L_p	56 mm
Radius of all the array elements, Wr	0.8 mm
Placement of parasitic elements from active element, d	62.5 mm

III. OPERATION OF THE SPA ANTENNA

The performance of parasitic arrays is governed by the mutual coupling amongst the array elements. The current distribution along the antenna elements in proximity of each other, changes as a result of the mutual coupling. Hence the antenna performance attributes, such as gain and input impedance, also change. The mutual coupling effect between antenna elements [17] depends on:

- Radiation characteristics of the individual element;
- Relative spacing between the elements;
- Orientation of each element;
- Feed position and magnitude of the array elements;
- Antenna type; and
- Operational frequency.

An analytical method such as the induced electromotive-force (EMF) can be used to obtain the performance attributes of the mutually coupled antenna system [17]. To obtain the SPA antenna performance, the induced EMF method is used to solve the self and mutual impedances [17], [18]. Thereafter, the currents along each antenna element can be computed, and then the radiation pattern and other performance attributes of the array are obtained. The solution of the induced EMF method depends on the length of the elements (L), the thickness of the elements (Wr) and the spacing in between the elements (d) [17], [18]. The operational frequency (f) also constitutes to the solution of the induced EMF method.

The antenna performance, in terms of gain and input impedance, is dependent on the currents along all the array elements, which are in turn dependent on the structural parameters. The self and mutual impedances of the antenna array are dependent on the array dimensions (structural parameters).

This inter-relationship between the characteristics (performance attributes) of the SPA antenna and the structural parameters is helpful in investigating how the variations in the antenna dimensions affect the response and performance of the SPA antenna. Variations in the structural parameters of the SPA antenna are assumed to model random error in the system.

IV. MONTE CARLO SIMULATION

To develop and fabricate multiple antenna prototypes is expensive and time consuming; therefore sensitivity analysis in this paper is carried out numerically using MATLAB. In this work, the numerical model is based on electromagnetic and statistical simulations.

If an assumption is made that: all the variations in the structural parameters of the SPA antenna are independent and random, a method such as the *Monte Carlo* simulation, can be used to predict the system behaviour. Monte Carlo simulation is used for repetitively evaluating the output of a deterministic model using sets of random numbers as inputs [16], [19]. The Monte Carlo simulation method is one of many methods for analyzing propagation of uncertainties, where the goal is to determine the sensitivity, performance, or reliability (tolerance) of the modelled system [19].

With the use of the induced EMF method, each structural parameter may be considered independently from the others. The errors in the structural parameters of the SPA antenna will lead to errors in the solutions of the Induced EMF method, and thus also to some uncertainty in the antenna gain and impedance match. Fig.2 below illustrates the error propagation due to the variations in structural parameters, in determining the SPA antenna gain and impedance.

We therefore assume that, the error for the system is an aggregated (total) error of the individual errors, as follows:

$$err_t = \sqrt{\sum_{n=1}^{N} (err_n)^2}, \quad (1)$$

where err_n is the error in a specific step in solving for the antenna performance attributes as illustrated in Fig. 2. We assume a total of N steps; err_t is the total error in the system for the antenna performance attributes (gain and input impedance).

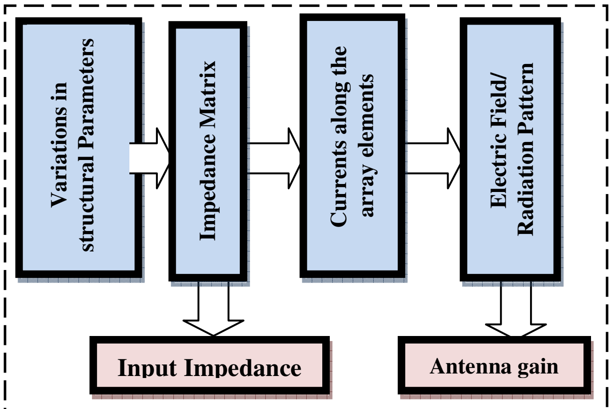

Fig. 2: Error propagation for the antenna performance measure.

Random errors in a measurement data can be easily presented using [12]:

$$y_{ij} = \mu_j + \xi_{ij}, \quad (2)$$

where y_{ij} is the i^{th} repeated measurement of the j^{th} variable in the function, for $i = 1,2,\ldots, m$ and j = $1,2,\ldots, n$. μ_i is the estimated average value of the variable y_{ij}. ξ_{ij} is the measurement error of the i^{th} replicate observation on the j^{th} variable.

If an assumption is made that a set of errors ξ_{ij} is independent and equally probable, then, the result of a single observation can be presented as [12]:

$$\mathbf{x} = g(y_1, y_2, \ldots, y_m), \quad (3)$$

where $g(\mathbf{y})$ is a particular function of interest. The entries of vector $\mathbf{y}$ are defined by (2), which are the repeated measurements of one specific variable. Equation (3) indicates the sampling distribution of the resulting vector $\mathbf{x}$, and the randomly subjected variable $\mathbf{y}$, which is in turn also subjected to random variation. Therefore, the distribution properties of $\mathbf{x}$ depend on both the assumptions about the nature of the function $g(\mathbf{y})$ and $\mathbf{y}$ itself [12].

In our case, the function $g(\mathbf{y})$ represents all the SPA antenna equations for solving the antenna performance attributes including the gain and impedance match. The function would be the transfer function of the system model as exemplified in Fig. 3. Equations (2) and (3) enable the implementation of the Monte Carlo simulation method because several input parameters can be individually varied and fed into the system model as the inputs.

Monte Carlo simulation is well applicable in our case because for any given set of antenna dimensions, there is always a specific set of outputs obtained by using the induced EMF method. However, any change in the antenna dimensions will result in another set of outputs which is not linearly correlated to the prior set of outputs because of the mutual coupling effect. Therefore, a nonlinear but deterministic approach is considered in examining the influence of the variations in the antenna dimensions on the performance attributes.

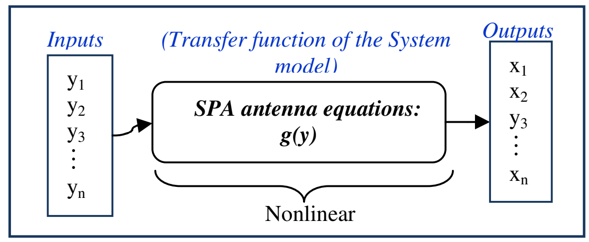

Fig. 3: Deterministic mapping of inputs and outputs of a system of integral equations.

V. SIMULATION RESULTS AND DISCUSSION

A. Simulation procedure

When using the Monte Carlo simulation method, we assume that, the worst total error (e.g. due to manufacturing) in the system can be 10% deviation in the specifications of all given structural parameters.

The simulation is carried out based on the following criteria:

- The antenna specifications in Table 1 are used as the mean values;
- All structural parameters are set to vary with 0.001% to10% from the mean values; allowing all structural parameters to vary with the same percentage error. The percentage error is assumed to be the total error in the system as variations in each structural parameter are independent and random;
- The structural parameters are randomly generated using the Gaussian distribution with the above-mentioned mean and standard deviation (based on the percentage error), and then used an input to the electromagnetic simulation. In this case, we assume variations in each structural parameter are independent from variations in the other structural parameters, although the percentage error is the same;
- About 10 000 samples (variations) for each structural parameter are generated;
- A deterministic computation on the inputs using (2) is performed, as illustrated in Fig. 3; and
- The simulation results are obtained and analyzed to determine the overall impact of the total error in the structural parameter of the SPA antenna gain and impedance match (return loss).

To evaluate the effects of the total error (as a result of variations in all parameters), we assume that all structural parameters vary independently although with the same percentage limit. This approach is used to avoid the computational complexity if we consider a case of individual parameter varying at its own limit regardless of the percentage error in other the structural parameters. Even though the same percentage error for all parameters is assumed, errors of individual structural parameters are independent and equally probable.

Fig. 4 indicates the effects of the overall variation in the structural parameters per defined percentage error in the parameters. A logarithmic scale is chosen to easily study the curve below the 1% error limit. From Fig. 4, the curve can be divided into two regions. *Region 1* is the increase in error from almost 0% to about 2%. In this region, the impact on the gain is significantly increasing with the increase in the percentage error in the structural parameters. In the second region (*Region 2*), from 2 % to 10%, increase in the error in the structural parameters does not have rapid effect (although more significant) on the antenna gain. The graph flattens in this area to confirm a slow change in the antenna gain deviation from the mean value.

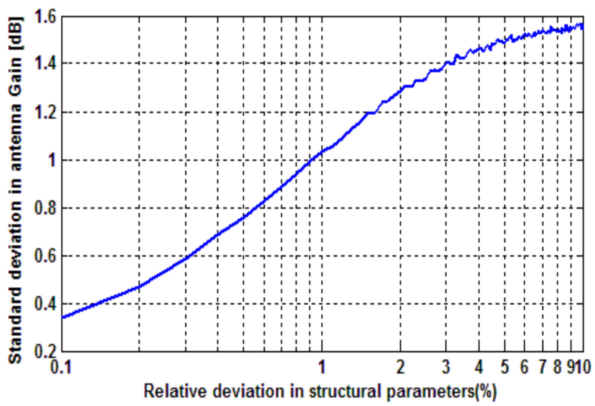

Fig. 4: Standard deviation in the antenna gain per logarithmic scale of the total error in the structural parameters.

The input impedance of the antenna can be translated into return loss in order to measure the effectiveness of the impedance match of the antenna and the transmission line. Assuming the transmission line impedance $Z_0 = 50\ \Omega$, the influences of the error in the structural parameters of the SPA antenna on the impedance match are calculated and presented in Fig. 5, which illustrate the impact of maximum percentage error in the structural parameters on the change in return loss (impedance match).

Unlike in the case of gain, the effects of variations in the structural parameters are even more significant on the return loss for minor errors. With about 1% error in the structural parameters, the return loss changes with a standard deviation of about 4.5 dB, while for the antenna gain the standard deviation is about 1 dB. For substantial errors (greater than 1%), the overall impact on the antenna impedance match is dynamically varying, with a deviation of about 5 dB.

It can be noticed that the overall effect of variations in individual parameters can be significant for the given antenna specification. The figures above, Fig. 4 and Fig. 5 demonstrate a situation where even minor variations (percentage error < 1%) result in the antenna performance greatly deviating from the mean values.

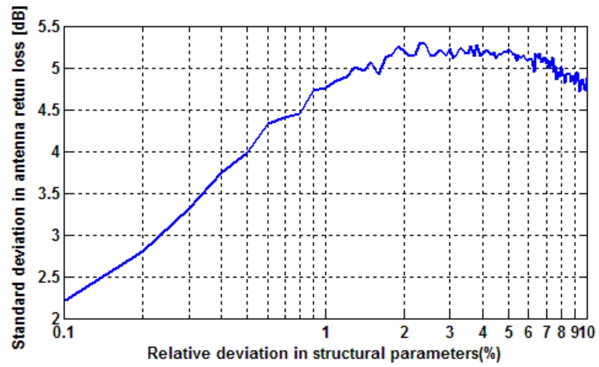

Fig. 5: Standard deviation in the antenna return loss (impedance match) per logarithmic scale of the % limit of the total error

The results can be generally summarized in two ways. Firstly, the results reveal that, the given antenna dimensions and the configuration used (one parasitic open-circuited and the other three short-circuited) may not be best for the antenna operating at the 2.4 GHz. This observation was also reported in [6], and the system dimensions were said to have been assumed, and not optimized. For optimized antenna geometry, deviations in both the antenna gain and impedance match are expected to be minimal for a larger range of errors.

On the other hand, if the errors are assumed to be due to manufacturing, we realize that the impact on both gain and input impedance is higher in *Region 1* (for errors less than 1%), therefore the possibility of low-manufacturing-cost of the SPA antenna may be affected. Minor errors in the system specifications may yield significant deviation in the antenna's performance. Hence, some precise techniques of manufacturing the antenna maybe needed if the very same specifications are to be used for the final antenna prototype.

VI. CONCLUSION AND FUTURE WORK

The Monte Carlo simulation has been used to numerically (based on the statistical and electromagnetic simulations) investigate the influence of variations (random errors) in the mechanical dimensions of a circular switched parasitic array SPA antenna operating at the 2.4 GHz. The results indicate that, even relatively small errors (as little as 1%) in the antenna's structural parameters have significant impact on the performance attributes (gain and impedance match) of the SPA antenna. The standard deviation in the impedance match is found to be an order of magnitude larger than the standard deviation in the antenna gain, for the same percentage error in the antenna dimensions.

The next stages of work may include comparison of the responses and performance of various optimized SPA antenna geometries, when experiencing variations in their structural parameters. Also, the results presented in this paper do not consider other variations in each array element, e.g. bending.

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