Characterisation of the flow regimes of arbitrary manoeuvre in absolute and relative frames

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Outline

Warning: this is a theory paper

- Context
- Background
- Some of the CFD background
- Computational background
- Theoretical background
- In the relative frame, in order to investigate relative size of terms,
- Continuity equation
- Momentum equation
- Energy equation

Outline II

- Dimensionless constants
- A question: longitudonal independence in rockets?
- Some illustrations
- Conclusions
- Further work

Context

- Engineering tools useless without…
- Engineering judgment which is based on…
- Understanding

Objectives: overall

- A formal framework for arbitrary manoeuvre
- CFD modelling of arbitrary manoeuvre
- Characterise dynamic loads in arbitrary manoeuvre

Objectives: specific

- Include acceleration terms $\ddot{r}$ and $\dot{\omega}$ in relative (body) frame formulation
- Include energy equation
- Find dimensionless numbers that are useful
- And thereby build the next step in the programme

Note: can only characterise linear behaviour in this way; nonlinear behaviour needs models

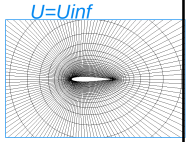
Background

- Batchelor, Greenspan, Landau and Lifshitz
- Directed largely at understanding atmospheric flows and waves
- Axial turbines
- Rothalpy and constant ω behaviour
- Flight dynamics
- The aims: bring this old news into CFD of arbitrary manoeuvres

CFD background

- Moving grids:
  - Chimera overset grids
  - Arbitrary Lagrangian Eulerian, ALE
  - Constant rotation ω: turbines and compressors
  - Small perturbations: aeroelasticity
  - Small perturbations: dynamic derivatives
  - Relative frame terms:
    - Roohani and Skews *2007…2011*
- In the inertial frame:
  - Inoue *et al.*

U=Uinf



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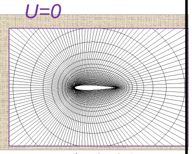


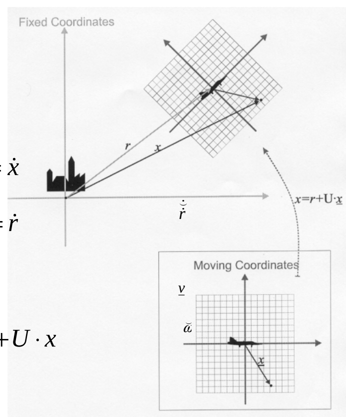
Theory Background to present work

- Transformation between frames moving with constant relative velocity is trivial: Galilean
  - Transformation between frames with relative acceleration is subject of present programme
- Löfgren
  - General formulation of transforms in 4-space between inertial and relative frames
  - Invariants in transformation
- Forsberg
  - Löfgren's formulation to simpler formulation
  - Numerical implications of inertial and relative frames
  - Stability and convergence in inertial and relative frames
- Forsberg et al. 2009
  - Implementation, validation and test cases

The parameters

Σ inertial frame: absolute

- Position vector of fluid element x
- Fluid velocity $v = \dot{x}$
- Position of *O´* r
  Velocity of *O´* $u = \dot{r}$
- Rotation vector of *Σ´*relative to *Σ* ω
- *Rotational transform* $\underline{x} = r + U \cdot x$

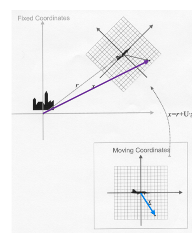


Vectors and transforms

	Σ inertial frame: absolute	***Σ´body frame: relative***
Position vector of fluid element	x	$\underline{x}$
Fluid velocity	$v = \dot{x}$	$\underline{v} = \dot{\underline{x}}$
Position of *O´* / Velocity of *O´*	r, $u = \dot{r}$	$\underline{r}$, $\underline{u} = \dot{\underline{r}}$

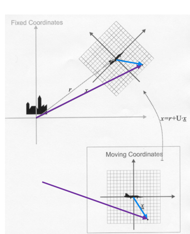
- *transform*

$$\underline{x} = r + U \cdot x$$
$$\frac{\partial U}{\partial t} = U\left(\frac{\partial}{\partial t} + \omega \times\right)$$
$$U^t = U^{-1}$$



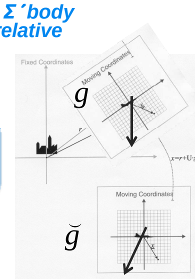
Vectors interpreted in the other frame

	Seen in Σ inertial frame: absolute		***Seen in Σ´body frame: relative***
vector	a	*as seen in Σ' is interpreted as* →	$\breve{a}$
	x	*as seen in Σ' is interpreted as* →	$\breve{x}$
	$\hat{\underline{a}}$	← *as seen in Σ' is interpreted as*	$\underline{a}$
	$\hat{\underline{x}}$	← *as seen in Σ' is interpreted as*	$\underline{x}$



Gravity vector interpreted in the other frame

	Seen in Σ inertial frame: absolute		***Seen in Σ´body frame: relative***
vector	g	*as seen in Σ' is interpreted as* →	$\breve{g}$



Scalars and intrinsic variables

Absolute, inertial

- ρ
- p
- T
- μ
- ν
- κ
- σ
- τ

Stress tensors are dependant on velocity

Relative: Conserved

- ρ
- p
- T
- μ
- ν
- κ
- $\breve{\sigma}$
- $\breve{\tau}$

- Is entropy S conserved?

Why is this notation so complex?

Why is it in the least important?

- It distinguishes in detail the frame transforms
- And provides a general framework

The general equation in conservation form for a conserved intrinsic quality *a*

Relative

$$\frac{\partial}{\partial t}(\rho a)+\nabla\cdot(\rho a\underline{v}+\breve{F}_a)=Q_a$$

Mass conservation: the equation of continuity

Relative

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\underline{v})=0$$

Momentum conservation

Relative

$$\frac{\partial}{\partial t}(\rho\underline{v})+\nabla\cdot(\rho\underline{v}\otimes\underline{v}+pI-\breve{\tau})=-\rho\ddot{\breve{r}}-\rho\dot{\breve{\omega}}\times\underline{x}-2\rho\breve{\omega}\times\underline{v}-\rho\breve{\omega}\times(\breve{\omega}\times\underline{x})+\rho\breve{g}$$

Energy conservation

Relative

$$E=e+\frac{\|\underline{v}+\underline{u}\|^2}{2}$$

$$a=E,\breve{F}_a=p\breve{v}-\breve{\tau}\cdot\breve{v}-\kappa\nabla T,Q_a=q_H+\breve{g}\cdot\breve{v}$$

$$\frac{\partial}{\partial t}(\rho E)+\nabla\cdot(\rho E\underline{v}+p\underline{v}-\breve{\tau}\cdot\underline{v}+p\underline{u}-\breve{\tau}\cdot\underline{u}-\kappa\nabla T)=q_H+\rho\underline{v}\cdot\breve{g}+\rho\dot{\breve{r}}\cdot\breve{g}+\rho\breve{\omega}\times\breve{g}$$

Generalised rothalpy *E**

Relative

$$E^* = e + \frac{\|\underline{v}\|^2}{2} - \frac{\|\underline{u}\|^2}{2}$$

$$\frac{\partial}{\partial t}(\rho E^*) + \nabla\cdot(\rho E^*\underline{v} + p\underline{v} - \kappa\nabla T)$$
$$= q_H + \rho\dot{\breve{r}}\cdot\ddot{\breve{r}} + \rho\dot{\breve{r}}\cdot\dot{\breve{\omega}}\times\underline{x}$$
$$+ \rho\ddot{\breve{r}}\cdot\dot{\breve{\omega}}\times\underline{x} + \rho\dot{\breve{\omega}}\times\underline{x}\cdot\breve{\omega}\times\underline{x}$$
$$+ \rho\underline{v}\cdot\ddot{\breve{r}} + \rho\underline{v}\cdot\dot{\breve{\omega}}\times\underline{x}$$
$$+ \rho\underline{v}\cdot\breve{g} + \rho\dot{\breve{r}}\cdot\breve{g} + \rho\breve{\omega}\cdot\breve{g}$$

1. First objective achieved…

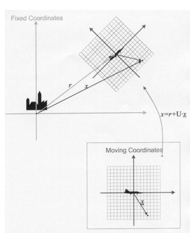
- Write out the equations including
- *g* and
- Viscous effects;
- And write out the energy equation in the relative frame.

Why?

- Now we can look at physical effects.

2. Next: find useful dimensionless constants for the momentum equation

- Several assumptions and notes:
- First: comparison to convective effects
- Only linear effects are identified this way
- For the present, single-scale problems are written; but most *r* scales will differ from *x* scales
- FYSA, Therefore, some gross simplifications



Momentum equation: typical scales

$$v = Uv_1,$$
$$x = Lx_1, \nabla = L^{-1}\nabla_1,$$
$$\omega = \Omega\omega_1, t = \Omega^{-1}t_1,$$
$$p = p_t p_1, \rho = \rho_t\rho_1, \tau = \nu\tau_1,$$
$$r = r_t r_1,$$

$$\frac{\partial}{\partial t}(\rho\underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v} + pI - \breve{\tau}) = -\rho\ddot{\breve{r}} - \rho\dot{\breve{\omega}}\times\underline{x} - 2\rho\breve{\omega}\times\underline{v} - \rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + \rho\breve{g}$$

$$[\frac{L}{t_t U}]\frac{\partial}{\partial t}(\rho\underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v}) + [\frac{p_t}{\rho_t U^2}]\nabla\cdot pI - [\frac{\nu L}{U}]\nabla\cdot\breve{\tau}$$

$$= -[\frac{L\ddot{\breve{r}}_t}{U^2}]\rho\ddot{\breve{r}} - [\frac{L^2\dot{\breve{\Omega}}}{U^2}]\rho\dot{\breve{\omega}}\times\underline{x} - [2\frac{L\Omega}{U}]\rho\breve{\omega}\times\underline{v} - [\frac{L^2\Omega^2}{U^2}]\rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + [\frac{L\|g\|}{U^2}]\rho\frac{\breve{g}}{\|g\|}$$

- The formal structure underlying the simulations of Roohani and Skews 2007…

$$\frac{\partial}{\partial t}(\rho\underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v} + pI - \breve{\tau}) = -\rho\ddot{\breve{r}} - \rho\dot{\breve{\omega}}\times\underline{x} - 2\rho\breve{\omega}\times\underline{v} - \rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + \rho\breve{g}$$

Extract meaning from models, term by term Strouhal

$$[\frac{L}{t_t U}]\frac{\partial}{\partial t}(\rho\underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v}) + [\frac{p_t}{\rho_t U^2}]\nabla\cdot pI - [\frac{\nu L}{U}]\nabla\cdot\breve{\tau}$$

$$= -[\frac{L\ddot{\breve{r}}_t}{U^2}]\rho\ddot{\breve{r}} - [\frac{L^2\dot{\breve{\Omega}}}{U^2}]\rho\dot{\breve{\omega}}\times\underline{x} - [2\frac{L\Omega}{U}]\rho\breve{\omega}\times\underline{v} - [\frac{L^2\Omega^2}{U^2}]\rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + [\frac{L\|g\|}{U^2}]\rho\frac{\breve{g}}{\|g\|}$$

- Strouhal number: typical temporal behaviour

$$St^{-1} = \frac{L}{t_t U}$$

Euler

$$[\frac{L}{t_t U}]\frac{\partial}{\partial t}(\rho \underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v}) + [\frac{p_t}{\rho_t U^2}]\nabla\cdot pI - [\frac{\nu L}{U}]\nabla\cdot\breve{\tau}$$

$$= -[\frac{L\ddot{\breve{r}}_t}{U^2}]\rho\ddot{\breve{r}} - [\frac{L^2\dot{\breve{\Omega}}}{U^2}]\rho\dot{\breve{\omega}}\times\underline{x} - [2\frac{L\Omega}{U}]\rho\breve{\omega}\times\underline{v} - [\frac{L^2\Omega^2}{U^2}]\rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + [\frac{L\|g\|}{U^2}]\rho\frac{\breve{g}}{\|g\|}$$

- Euler number

$$Eu^{-1} = \frac{p_t}{\rho_t U^2}$$

Reynolds: translational viscous effects

$$[\frac{L}{t_t U}]\frac{\partial}{\partial t}(\rho \underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v}) + [\frac{p_t}{\rho_t U^2}]\nabla\cdot pI - [\frac{\nu L}{U}]\nabla\cdot\breve{\tau}$$

$$= -[\frac{L\ddot{\breve{r}}_t}{U^2}]\rho\ddot{\breve{r}} - [\frac{L^2\dot{\breve{\Omega}}}{U^2}]\rho\dot{\breve{\omega}}\times\underline{x} - [2\frac{L\Omega}{U}]\rho\breve{\omega}\times\underline{v} - [\frac{L^2\Omega^2}{U^2}]\rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + [\frac{L\|g\|}{U^2}]\rho\frac{\breve{g}}{\|g\|}$$

- Reynolds number: typical viscous behaviour, different normalisation

$$\mathrm{Re}^{-1} = \frac{\nu L}{U}\frac{1}{L^2}$$

Ekman: rotational viscous effects

$$[\frac{L}{t_t U}]\frac{\partial}{\partial t}(\rho \underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v}) + [\frac{p_t}{\rho_t U^2}]\nabla\cdot pI - [\frac{\nu L}{U}]\nabla\cdot\breve{\tau}$$

$$= -[\frac{L\ddot{\breve{r}}_t}{U^2}]\rho\ddot{\breve{r}} - [\frac{L^2\dot{\breve{\Omega}}}{U^2}]\rho\dot{\breve{\omega}}\times\underline{x} - [2\frac{L\Omega}{U}]\rho\breve{\omega}\times\underline{v} - [\frac{L^2\Omega^2}{U^2}]\rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + [\frac{L\|g\|}{U^2}]\rho\frac{\breve{g}}{\|g\|}$$

- Ekman number: development of boundary layers and end-wall viscous phenomena in rotationally dominated flows

$$U = \Omega L$$

$$Ek^{-1} = \frac{\nu}{\Omega L^2}$$

Translational Acceleration of the frame

$$[\frac{L}{t_t U}]\frac{\partial}{\partial t}(\rho \underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v}) + [\frac{p_t}{\rho_t U^2}]\nabla\cdot pI - [\frac{\nu L}{U}]\nabla\cdot\breve{\tau}$$

$$= -[\frac{L\ddot{\breve{r}}_t}{U^2}]\rho\ddot{\breve{r}} - [\frac{L^2\dot{\breve{\Omega}}}{U^2}]\rho\dot{\breve{\omega}}\times\underline{x} - [2\frac{L\Omega}{U}]\rho\breve{\omega}\times\underline{v} - [\frac{L^2\Omega^2}{U^2}]\rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + [\frac{L\|g\|}{U^2}]\rho\frac{\breve{g}}{\|g\|}$$

- Note the similarity to the gravitational term…

$$\frac{L\ddot{\breve{r}}_t}{U^2} \qquad \frac{Lg}{U^2}$$

Rotational Acceleration of the frame

$$[\frac{L}{t_t U}]\frac{\partial}{\partial t}(\rho \underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v}) + [\frac{p_t}{\rho_t U^2}]\nabla\cdot pI - [\frac{\nu L}{U}]\nabla\cdot\breve{\tau}$$

$$= -[\frac{L\ddot{\breve{r}}_t}{U^2}]\rho\ddot{\breve{r}} - [\frac{L^2\dot{\breve{\Omega}}}{U^2}]\rho\dot{\breve{\omega}}\times\underline{x} - [2\frac{L\Omega}{U}]\rho\breve{\omega}\times\underline{v} - [\frac{L^2\Omega^2}{U^2}]\rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + [\frac{L\|g\|}{U^2}]\rho\frac{\breve{g}}{\|g\|}$$

- Angular acceleration

$$\frac{L^2\dot{\breve{\Omega}}}{U^2}$$

Coriolis effects

$$[\frac{L}{t_t U}]\frac{\partial}{\partial t}(\rho \underline{v}) + \nabla\cdot(\rho\underline{v}\otimes\underline{v}) + [\frac{p_t}{\rho_t U^2}]\nabla\cdot pI - [\frac{\nu L}{U}]\nabla\cdot\breve{\tau}$$

$$= -[\frac{L\ddot{\breve{r}}_t}{U^2}]\rho\ddot{\breve{r}} - [\frac{L^2\dot{\breve{\Omega}}}{U^2}]\rho\dot{\breve{\omega}}\times\underline{x} - [2\frac{L\Omega}{U}]\rho\breve{\omega}\times\underline{v} - [\frac{L^2\Omega^2}{U^2}]\rho\breve{\omega}\times(\breve{\omega}\times\underline{x}) + [\frac{L\|g\|}{U^2}]\rho\frac{\breve{g}}{\|g\|}$$

- Rossby number:
- Ro<<1, rotational effects dominate the flow; Taylor-Proudman columns in inviscid, isentopic, incompressible flow

$$Ro^{-1} = 2\frac{L\Omega}{U}$$

Centrifugal effects

$$[\frac{L}{t_t U}]\frac{\partial}{\partial t}(\rho \underline{v}) + \nabla \cdot (\rho \underline{v} \otimes \underline{v}) + [\frac{p_t}{\rho_t U^2}]\nabla \cdot pI - [\frac{\nu L}{U}]\nabla \cdot \breve{\tau}$$

$$= -[\frac{L\dot{\ddot{r}}_t}{U^2}]\rho\dot{\ddot{r}} - [\frac{L^2\dot{\breve{\Omega}}}{U^2}]\rho\dot{\breve{\omega}} \times \underline{x} - [2\frac{L\Omega}{U}]\rho\breve{\omega} \times \underline{v} - [\frac{L^2\Omega^2}{U^2}]\rho\breve{\omega} \times (\breve{\omega} \times \underline{x}) + [\frac{L\|g\|}{U^2}]\rho\frac{\breve{g}}{\|g\|}$$

$$\frac{L^2\Omega^2}{U^2}$$
if
$$U = R\Omega,$$

- Centrifugal effects, e.g. wake curvature, are on scale $\frac{R^2}{L^2}$

our future through science

Gravitational effects

$$[\frac{L}{t_t U}]\frac{\partial}{\partial t}(\rho \underline{v}) + \nabla \cdot (\rho \underline{v} \otimes \underline{v}) + [\frac{p_t}{\rho_t U^2}]\nabla \cdot pI - [\frac{\nu L}{U}]\nabla \cdot \breve{\tau}$$

$$= -[\frac{L\dot{\ddot{r}}_t}{U^2}]\rho\dot{\ddot{r}} - [\frac{L^2\dot{\breve{\Omega}}}{U^2}]\rho\dot{\breve{\omega}} \times \underline{x} - [2\frac{L\Omega}{U}]\rho\breve{\omega} \times \underline{v} - [\frac{L^2\Omega^2}{U^2}]\rho\breve{\omega} \times (\breve{\omega} \times \underline{x}) + [\frac{L\|g\|}{U^2}]\rho\frac{\breve{g}}{\|g\|}$$

- buoyancy term in comparison to pressure effects in hydrostatics

$$\frac{Lg}{U^2}\frac{\delta p}{\rho}$$

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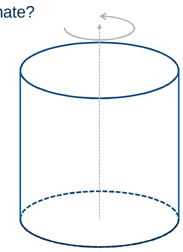
3. A question: can Taylor columns exist in missile combustion chambers?

- When do rotational effects dominate?



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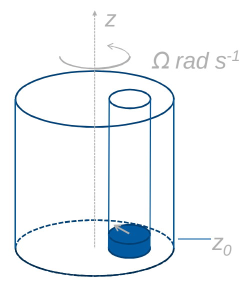
Taylor-Proudman theorum

- Constant angular velocity Ω
- Rossby number
  Ro = U/2ΩL
- For Ro<<1, rotational effects dominate convection
- For incompressible, inviscid flow,
- ∂/∂z=0
- An obstacle A which is moved and generates streamlines at z_0
- …generates identical streamlines at all z



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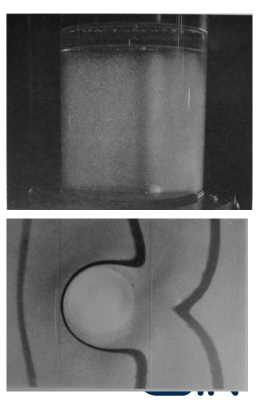
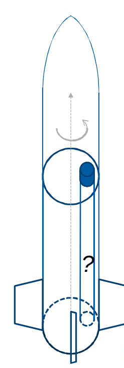
- Accomplished by slight spin-up or spin-down
- Theory, Proudman 1916
- Experiment, Taylor 1917



our future through science

- Could Taylor columns exist in internal flow in missiles and rockets?
- What influence can be predicted on external flow?
- Progressive assumptions



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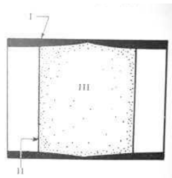
Rockets or missiles considered as…

- Closed cylinders,
- Incompressible,
- Almost rigid rotation,
- With Rossby number *Ro* characterising rotational dominance
- Viscous effects can be characterised [Greenspan] by times scales related to the
- Ekman number:



[Greenspan] spin-up from rest:
I Ekman boundary layer
II front,
III almost quiescent core

- t~1, development of viscous boundary layers (I),
- $t \sim \Omega^{-1} Ek^{-1/2}$, spin-up time (II, III),
- $t \sim \Omega^{-1} Ek^{-1/2}$, decay of residual viscous effects
- Take ν for dry air for the present

Typical values

- For rigid rotation, Ro ~ ½
- Will viscous effects dominate Taylor columns?
- Apache and Cajun sounding rockets
- Uselton and Carman ADC 1969
- $\Omega \sim$ 1.5 to 32 rads^{-1}, radius 59mm
- Highly manoeuvrable missiles,
- Marquardt, Lawrence and Lawrence, AEDC, 1998
- $\Omega \sim$ 100 rads^{-1}, radius r ~ 59 mm
- Unguided fin-stabilised artillery rockets, 122mm
- Khalil *et al.*, Egypt Armed Forces, 2009
- Muzzle $\Omega \sim$ 100 rads^{-1}

- For th

	Low spin [Uselton and Carman 1969]	Medium spin [Uselton and Carman 1969]	High spin [Khalil *et al.* 2009]
Spin rate Ω	1.5 rads^{-1}	32 rads^{-1}	100 rads^{-1}
Radius r	59 mm	59 mm	61 mm
Ro, rigid	.5	.5	.5
Ek	70×10^{-5}	3.4×10^{-5}	1.1×10^{-5}
Ekman layer time	.67 s	.03 s	.01 s
Spin up time	24.9 s	5.4 s	3.1 s
Residual viscous effect time	930 s	930 s	930 s
Comment	Taylor columns possible in this approx	Taylor columns possible in this approx	Ekman layer established fast, but Taylor columns possible i.t.a.; burn time is 1.8 s

Angular acceleration and translational acceleration

- Khalil *et al.* 2009
- Fin stabilised artillery rockets
- Measurements of translational acceleration, spin rate available
- $\dot{\omega}$ ~ 100 rads^{-1}, ω ~ 100 rads^{-1}

$$\frac{L^2\dot{\Omega}}{U^2} \approx \frac{\dot{\Omega}}{\Omega^2} \approx 0.01$$

- Linear approximation indicates low angular acceleration effects…
- But we are aware of the vortex interactions through CFD
- $\dot{r}$ ~500 ms-2 for thrust of 23600N,
- $\frac{L\dot{r}_t}{U^2}$ ~ 2.1, a significant factor even in these terms:
  translational acceleration likely to have significant effect

Translational acceleration
Roohani and Skews, 2007 and 2011

- Biconvex, NACA0012, NACA2412 and RAE2812 airfoils

  $\dot{r}$ = 1041ms^{-2}
  For transonic cases,
- $\frac{L\dot{r}_t}{U^2}$ ~ 0.012
- But very significant changes are experienced – and these are non-linear, due to shock position and shape
- Subsonic cases: U~100 ms^{-1}
- $\frac{L\dot{r}_t}{U^2}$ ~ 0.1, and significant changes in linear range should be apparent

Conclusions

- 1. the energy equation is expressed in terms of translations and rotational acceleration,
- 2. the generalised enthalpy equation is similarly expressed,
- 3. but the meaning of these is still to be explored
- 4. dimensionless constants for momentum changes are re-derived,
- 5. But translational acceleration needs reconsideration in the light of numerical experiments,
- And rotational indications by Ro and Ek are that Taylor columns in rockets need consideration in terms of heat transfer, boundary conditions and compressibility

Further work

- Compressibility
- Thermodynamics: is entropy conserved in frame transformation?
  - Express energy equation in terms of T
  - Derive dimensionless numbers
- Taylor columns:
  - Compressibility
  - Boundary conditions
  - Heat transfer
- Shocks:
  - Do Rankine-Hugoniot relations transform?

Further work II

- Boundary conditions on accelerating walls
  - Boundary layer formation
- Turbulence
  - How do we deal with numerical turbulence models?
  - Is it appropriate to apply classic turbulence models even in the absolute frame?
- Perturbations
  - Rossby waves
  - Orr-Sommerfeld and transition

¿questions?