Introducing a moisture scheme to a nonhydrostatic sigma coordinate model

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Introduction

A nonhydrostatic sigma coordinate model (NSM) is currently being developed at the Council for Scientific and Industrial Research (CSIR), using the equation set of Engelbrecht et. al. (2007), for purposes of simulating weather at spatial resolutions where the hydrostatic approximation is not valid. The aim of this study is to introduce a moisture scheme to the NSM, for the explicit simulation of moist convection.

Models that simulate clouds explicitly use microphysical parameterisations which are grouped into bulk and bin approaches (Stensrud, 2007). Bulk approaches use a specified function for the particle size distributions and generally predict the particle mixing ratio (Rutledge and Hobbs, 1983). The particle size distribution is usually approximated by the inverse exponential distribution, in this study we follow the same approach. A bin approach does not use a specified function for the particles distribution. It divides the particle distribution into a number of finite size and categories. This division of particle distribution into numerous bins requires much larger memory and computational capabilities, and poor knowledge of ice phase physics hampers the accurate representation of evolving ice particle concentrations. Therefore bin methods are employed in a few research models (Stensrud, 2007).

The Equation Set

In this study we introduce a bulk scheme because we would like to use this model for operational forecasting in the near future. We follow closely the scheme used by Rutledge and Hobbs (1983) and Khairoutdinov and Randall (2003).

$$\frac{Du}{Dt} - fv + \frac{\partial \varphi}{\partial x} - \sigma \frac{\partial \varphi}{\partial \sigma} \frac{\partial \ln p_s}{\partial x} = 0 \quad (1)$$

$$\frac{Dv}{Dt} + fu + \frac{\partial \varphi}{\partial y} - \sigma \frac{\partial \varphi}{\partial \sigma} \frac{\partial \ln p_s}{\partial y} = 0 \quad (2)$$

$$\frac{1}{g} \frac{D}{Dt}\left(\frac{R_m \omega T}{p}\right) + g + \frac{p}{p_s} \frac{g}{R_m T} \frac{\partial \varphi}{\partial \sigma} = 0$$
$$(3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \dot{\sigma}}{\partial \sigma} + \frac{D \ln p_s}{Dt} = 0 \quad (4)$$

$$\frac{DT}{Dt} - \kappa \frac{\omega T}{p} = \frac{S_h}{c_p} \quad (5)$$

$$\frac{Dq_i}{Dt} = S_i \quad (6)$$

$$\frac{Dq_j}{Dt} = S_j + FALLOUT \quad (7)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial}{\partial \sigma}\left[s^2 \frac{\partial \phi}{\partial \sigma}\right]$$

$$-2\sigma\left[\frac{\partial \ln p_s}{\partial x}\left(\frac{\partial^2 \phi}{\partial x \partial \sigma}\right) + \frac{\partial \ln p_s}{\partial y}\left(\frac{\partial^2 \phi}{\partial y \partial \sigma}\right)\right]$$

$$+\frac{\partial}{\partial \sigma}\left(\sigma^2 \frac{\partial \phi}{\partial \sigma}\right)\left[\left(\frac{\partial \ln p_s}{\partial x}\right)^2 + \left(\frac{\partial \ln p_s}{\partial y}\right)^2\right]$$

$$-\frac{\sigma}{p_s} \frac{\partial \phi}{\partial \sigma}\left[\frac{\partial^2 p_s}{\partial x^2} + \frac{\partial^2 p_s}{\partial y^2}\right] =$$

$$2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\frac{\partial}{\partial \sigma}\left(\frac{\Omega p}{p_s}\right) -$$

$$\frac{2}{p_s}\left[\frac{\partial u}{\partial \sigma} \frac{\partial}{\partial x}(p\Omega) + \frac{\partial v}{\partial \sigma} \frac{\partial}{\partial y}(p\Omega)\right]$$

$$+2\left(\frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial v}{\partial x} \frac{\partial u}{\partial y}\right)$$

$$+2\sigma\left[\frac{\partial \ln p_s}{\partial x}\left(\frac{\partial u}{\partial y} \frac{\partial v}{\partial \sigma} - \frac{\partial v}{\partial y} \frac{\partial u}{\partial \sigma}\right)\right]$$

$$+2\sigma\left[\frac{\partial \ln p_s}{\partial y}\left(\frac{\partial v}{\partial x} \frac{\partial u}{\partial \sigma} - \frac{\partial u}{\partial x} \frac{\partial v}{\partial \sigma}\right)\right]$$

$$+ \sigma f \left[\frac{\partial \ln p_s}{\partial y} \frac{\partial u}{\partial \sigma} - \frac{\partial \ln p_s}{\partial x} \frac{\partial v}{\partial \sigma} \right]$$

$$+ f \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

$$- u \frac{\partial f}{\partial y} - \frac{\partial}{\partial \sigma} \left(sg - \frac{p}{p_s} \Omega^2 \frac{1}{\gamma} \right)$$

$$- \frac{\partial}{\partial \sigma} \left(\frac{S_h p \Omega}{p_s T c_p} \right) - \frac{\partial}{\partial \sigma} \left(\frac{R_v q_v \Omega \omega}{p_s c_p} \right)$$

$$- \frac{\partial}{\partial \sigma} \left(\frac{\Omega R_v p}{R_m p_s} \frac{Dq_v}{Dt} \right) \quad (8)$$

Equations 1 to 3 are the x, y, and sigma coordinates momentum equations. The vertical momentum equation contains the gas constant of a mixture of dry air and moisture. The thermodynamic energy equation includes heating or cooling by latent heat release or absorption.

Equation 6 is the water continuity equation for predicting the mixing ratios of water vapour, cloud water, and ice. Cloud water and ice particles are assumed to have one size throughout the cloud. The right hand side represents microphysical sources and sinks of the water particles. Condensation is for example a source for cloud water and a sink for water vapour. Ice melting is a sink for ice and a source for cloud water. Equation 7 is for rain water and snow which are assumed to have an inverse exponential size distribution. Both have a fall speed and that is indicated by FALLOUT in the equation.

The approximations in the model introduced to obtain a quasi-elastic equation set requires that a computationally expensive diagnostic geopotential perturbation (elliptic) equation (equation 8) be solved at each time step. The last three terms in equation 8 are a consequence of moisture and the microphysics processes associated with it.

Figure 1 shows potential temperature as simulated for a moist bubble in which condensation was allowed to take place. The bubble with moisture is warmer and rises faster than a dryer one because of warming by latent heat release.
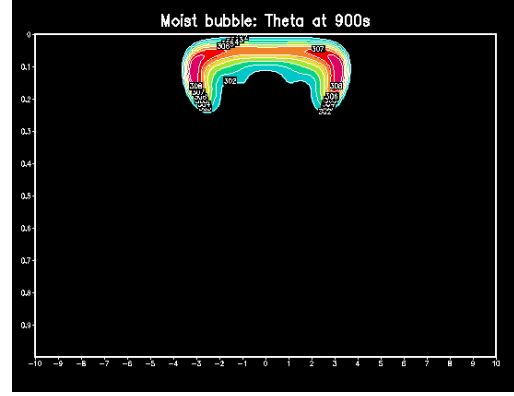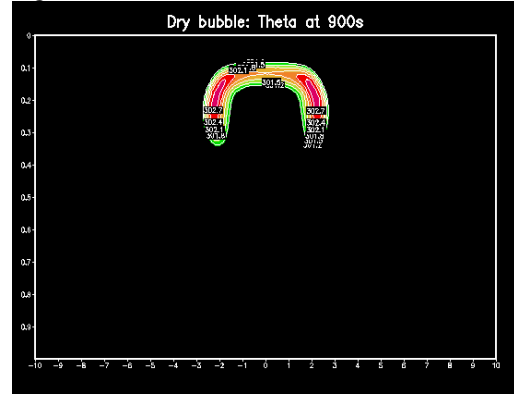

Figure1: Moist bubble theta simulation.

Figure 2: Dry bubble theta simulation

Way forward

The NSM will be used to simulate convection in an environment that is similar to the real atmosphere with different bulk microphysics schemes.

References
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