Superimposed chirped pulse parameter estimation based on the Extended Kalman Filter (EKF)

J.C. Olivier
Electrical, Electronics and Computer engineering
University of Pretoria
Pretoria, 0001, SA
Email: corne.olivier@up.ac.za

and

Defence, Peace, Safety and Security, CSIR
Pretoria, SA, 0001.

***Abstract*— An Extended Kalman Filter (EKF) is proposed to estimate the frequencies and chirp rate of multiple superimposed chirped pulses. The estimation problem is a difficult one, where Maximum Likelyhood methods are very complex especially if more than two pulses are superimposed. It is shown that for the EKF an arbitrary number of pulses can be tracked, and it will be shown that for the case of two superimposed pulses tracking is achieved after sufficient samples for minimum required frequency resolution. Mean Square Error (MSE) performance is shown to compare well to the Cramer-Rao bounds even for difficult cases where the number of samples is close to the inverse of the difference in normalized frequencies.**

I. Introduction

Chirp pulsed signals are common in many applications of active and passive Sonar as well as Radar and Electronic Warfare. Specifically for the case where the objective is security and surveillance, the receiver is concerned with the estimation of parameters such as the amplitudes, frequencies and chirp rates of multiple superimposed chirped pulses.

The superimposed chirp parameter estimation problem is a complex one and has received considerable attention in the literature. The Maximum Likelyhood (ML) approach was proposed in [1] where a rank-reduction method was used to get initial estimates after which the ML approach yielded the final estimates. The approach however is not guaranteed to find the global minimum of the ML function. In [2] a computationally complex method based on ML Monte Carlo importance sampling was proposed. Results for the case of two superimposed pulses were provided that show the methods ability to achieve the Cramer-Rao bounds above 3 dB signal to noise ratio (SNR) under additive white Gaussian noise. A Markov Chain Monte Carlo approach was proposed in [3], with results and complexity similar to that of [2].

Recently the Kalman filter has been proposed as a practical and low complexity solution to *single pulse* parameter estimation [4], [5], [6]. The Kalman filter is able to perform parameter estimation even if the amplitude and the rate of frequency change is time varying and the noise is non Gaussian [4]. In this paper it will be shown that the Extended Kalman filter (EKF) is able to estimate chirp pulse parameters for the case of superimposed signals with low complexity, even for cases where the length of the observed data is close to the inverse of the difference in the normalized frequencies.

The paper is organized as follows. Section 2 briefly derives the EKF for estimating the parameters of supersimposed chirp pulses. In section 3 numerical results are shown. Conclusions are presented in Section 4.

II. The Extended Kalman Filter for Chirp rate Estimation

The Extended Kalman Filter (EKF) is one of the so-called *analytic* approximations to the optimal recursive Bayesian estimator. It approximates (linearises) the nonlinear functions in the state dynamic and measurement models. Specifically the nonlinear functions in the process and measurement models are approximated by the first term in the Taylor series expansion. Assuming the random sequences v_{k-1} (process noise) and w_k (measurement noise) are mutually independent, zero mean white Gaussian with covariance $\mathbf{Q}_{k-1}$ and $\mathbf{R}_k$ respectively, then the time iterations are performed as [7]

$$\begin{aligned}\hat{\mathbf{x}}_{k|k-1} &= f_{k-1}(\hat{\mathbf{x}}_{k-1|k-1}) \\ \mathbf{P}_{k|k-1} &= \mathbf{Q}_{k-1} + \hat{\mathbf{F}}_{k-1}\mathbf{P}_{k-1|k-1}\hat{\mathbf{F}}_{k-1}^T \\ \hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(\mathbf{z}_k - \mathbf{h}_k(\hat{\mathbf{x}}_{k|k-1})) \\ \mathbf{P}_{k|k} &= \mathbf{P}_{k|k-1} - \mathbf{K}_k\mathbf{S}_k\mathbf{K}_k^T\end{aligned} \tag{1}$$

where

$$\begin{aligned}\mathbf{S}_k &= \hat{\mathbf{H}}_k\mathbf{P}_{k|k-1}\hat{\mathbf{H}}_k^T + \mathbf{R}_k \\ \mathbf{K}_k &= \mathbf{P}_{k|k-1}\hat{\mathbf{H}}_k^T\mathbf{S}_k^{-1}\end{aligned} \tag{2}$$

and $\hat{\mathbf{F}}_{k-1}$ and $\hat{\mathbf{H}}_k$ are the local linearization of nonlinear functions $\mathbf{f}_{k-1}$ and $\mathbf{h}_k$ respectively. These are defined as Jacobians evaluated at $\hat{x}_{k-1|k-1}$ and $\hat{x}_{k|k-1}$ respectively. I.e.

$$\begin{aligned}\hat{\mathbf{F}}_{k-1} &= [\nabla_{\mathbf{x}_{k-1}}\mathbf{f}_{k-1}^T(\mathbf{x}_{k-1})]^T|_{\mathbf{x}_{k-1}=\hat{x}_{k-1|k-1}} \\ \hat{\mathbf{H}}_k &= [\nabla_{\mathbf{x}_k}\mathbf{h}_k^T(\mathbf{x}_k)]^T|_{\mathbf{x}_k=\hat{x}_{k|k-1}}\end{aligned} \tag{3}$$

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where

$$\nabla_{\mathbf{x}_k} = \left[\frac{\partial}{\partial \mathbf{x}_k[1]}, \cdots, \frac{\partial}{\partial \mathbf{x}_k[n_x]}\right], \quad (4)$$

assuming there are n_x components in $\mathbf{x}_k$.

A. EKF Formulation for two superimposed chirp pulses

This section derives the EKF formulation for two superimposed Linear Frequency Modulated (LFM) pulses, while the formulation can be generalized to N pulses in a straightforward manner.

Using frequencies normalized to unity as is done in [2], the observations are generated by a process given by

$$z[k] = A_1 \exp^{j(\omega_1 k + \pi m_1 k^2)} + A_2 \exp^{j(\omega_2 k + \pi m_2 k^2)} + n_k \quad (5)$$

where A_i denote the unknown complex amplitudes including a phase shift, ω_i the unknown angular frequencies, and m_i the unknown chirp rate parameters. n_k is a random number drawn from a complex white Gaussian process with variance σ^2 and zero mean. Let us define a time varying frequency f_i to be tracked over time by the EKF. For a LFM pulse we may write f_i as

$$f_i[k] = f_i^0 + \frac{m_i k}{2}. \quad (6)$$

The observations given by (5) using the time varying frequencies are given by

$$z[k] = A_1 \exp^{j(2\pi f_1 k)} + A_2 \exp^{j(2\pi f_2 k)} + n_k. \quad (7)$$

This observational model is autoregressive (AR), since for the first pulse we may write

$$x_1^{k-1} = A_1 \exp^{j(2\pi f_1 (k-1))} = x_1^k exp^{-j\omega}. \quad (8)$$

Hence it can be deduced that

$$x_1^k = x_1^{k-1} exp^{j\omega} = x_1^{k-1} x_3^{k-1}, \quad (9)$$

where clearly $x_3^k = x_3^{k-1}$. The same holds for all the other pulses.

Thus for both pulses a state vector as a vector containing the discrete time k and the unknown parameters, we may write

$$\mathbf{x}^k = \begin{bmatrix} x_1^k \\ x_2^k \\ x_3^k \\ x_4^k \end{bmatrix} = \begin{bmatrix} x_1^{k-1} x_3^{k-1} \\ x_2^{k-1} x_4^{k-1} \\ x_3^{k-1} \\ x_4^{k-1} \end{bmatrix}. \quad (10)$$

Except for $x[1]$ and $x[2]$ the other parameters are time invariant, so that these should converge to fixed values. The chirp rate can be estimated from the slope of the frequency $f_i(k)$.

The initial conditions $\mathbf{x}_0$ required for guaranteed convergence are not well understood for the case where more than one pulse are superimposed. There are choices of initial conditions that will not lead to a valid solution, but these initial conditions are difficult to quantify analytically. In practice it was found that we may however identify invalid solutions by computing the autocorrelation function of the innovation γ [7] given by

$$\gamma_k = \mathbf{z}_k - \mathbf{h}_k(\mathbf{x}_k, \mathbf{w}_k). \quad (11)$$

A valid solution has an innovation autocorrelation that approximates a Dirac delta function [1]. If a solution is found to be invalid, a different initial condition can be tried until a valid solution is found. It was found that a small number random retries with random initial conditions yields good performance and that the longer the pulse are the less random tries are needed.

The EKF requires a Jacobian to locally linearise the nonlinearities in the state equation. The Jacobian has the following form:

$$\begin{bmatrix} x_3^k & 0 & x_1^k & 0 \\ 0 & x_4^k & 0 & x_2^k \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (12)$$

The observation system is linear with a matrix $H = [1, 1, 0, 0]$. The recursive equations to perform the estimations were given in a previous section.

III. Numerical results

In this section we study the performance of the EKF for the case of a finite chirp rate. In order to compare the EKF with the ML method using Monte Carlo Importance sampling as proposed by Kay [2], the simulation settings used in [2] was used for this case. These were as follows:

1) $f_1, f_2 = [0.3, 0.32]$
2) $m_1, m_2 = [0.001, 0.002]$
3) $A_1, A_2 = [1, 1]$ V
4) Pulse length $N = 50$ samples
5) sampling at Nyquist rate.

These settings represent a difficult case for any estimator. First of all the frequencies are closely spaced together, at the limit where separation will occur as the resolution is $\frac{1}{50} = 0.2$. Also the two pulses have equal amplitudes, the worst case scenario, and the chirp rate is fairly high. The results shown in Figure 1 indicate performance for the EKF relative to the CR bounds. It was found that performance rapidly improved (relative to the CR bounds) if N was increased and if the chirp rate was reduced. This may be understood by examination of the tracking of the instantaneous frequency over time as shown in Figure 2 (where pulse 2 has a small chirp rate and pulse 1 is not chirped). The tracker eventually achieves the theoretical tracking curve after about $\frac{2}{|f_1 - f_2|}$ samples, but for short pulses close to the minimum resolution $\frac{1}{|f_1 - f_2|}$ the tracking and hence the slope has not yet stabilized, leading to an increase in estimator variance in both the starting frequency f_i as well as the chirp rate m_i.

Results from the ML method with Monte Carlo importance sampling [2] indicates it achieves the CR exactly above a SNR of 3 dB and hence is able to outperform the EKF, but at the cost of more complexity. Also the EKF can be readily

[1] The prediction error is then random, i.e. has no structure

extended without increasing complexity significantly to handle more than two pulses which is not the case for the method in [2] where the number of Monte Carlo samples needed grows exponentially with the number of superimposed pulses.

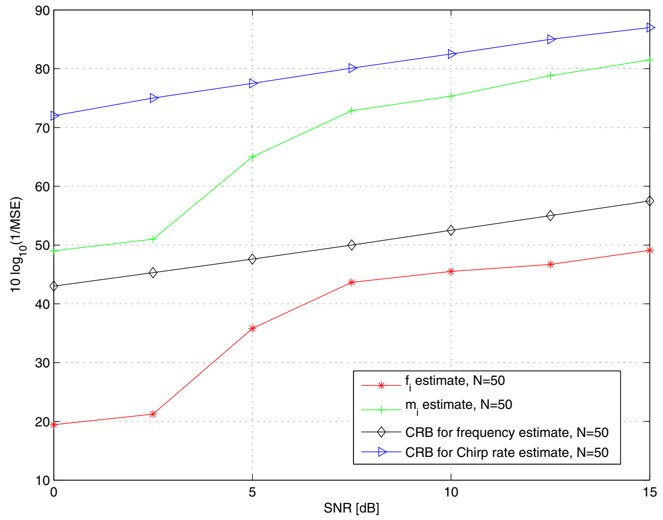

Fig. 1. The estimator performance for two superimposed pulses with a finite chirp rate.

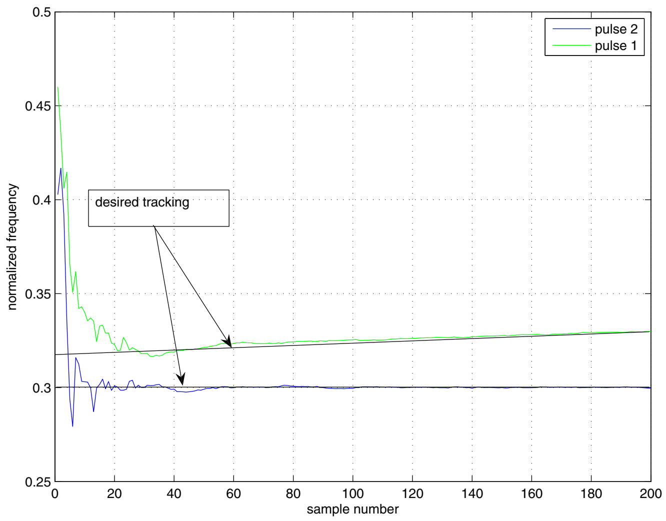

Fig. 2. The tracking performance of time of the EKF.

IV. Conclusions

An EKF was shown to be able to track the instantaneous frequencies of multiple superimposed chirped pulses. For the case of two superimposed pulses the EKF estimator tracks the instantaneous frequencies of the pulses after $\frac{2}{|f_1 - f_2|}$ samples from a random starting guess (without any prior knowledge). Extending the EKF formulation to more than two pulses is straightforward, and has moderate implication for the complexity of the recursive equations, which is not the case for methods based on the ML methods [1], [2], [3].

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