A Connectivity Model for Inter-working Multi-hop Wireless Networks.

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Abstract— In inter-working multi-hop wireless networks, establishing resilient connectivity between source-destination node pairs is a major issue. The issues of connectivity in multi-hop wireless networks have been studied. However these analyses focused on network connectivity in ad-hoc networks. Since the next generation of wireless networks will be inter-working, an understanding of connectivity as it applies to such networks is needed. Specifically, this research emphasizes that the connectivity of node pairs in inter-working multi-hop wireless networks can be evaluated based on the availability of radio links and communication routes. This paper presents an analytical study of the link and route availability in inter-working multi-hop wireless networks.

Index Terms-Availability, Connectivity, Inter-working, Multi-hop Wireless Networks.

I. INTRODUCTION

Connectivity is a fundamental property of any wireless network. Normally, in all networks, links are the basic element that ensures connectivity. In wired networks, links are readily provided by the communication cable and these links are stable and predictable. However, in wireless networks, links are provided by the air interface (wireless channel).

Generally, in wireless networks, nodes have to be within an appreciable distance of each other before a communication link can be established between them. Any node that is not within the recommended range is said to be out of the network. In single hop wireless networks, it is sufficient for each node to be within the transmission range of at least one of the centralized base stations in order to communicate with another node. In multi-hop wireless networks, if source-destination pairs are not within each other's transmission range, packets reach their destination nodes after some hops on nodes in between the source and destination. One of the advantages of multi-hop communications is that it ensures efficient spatial re-use.

In multi-hop wireless networks, the choice of the next hop depends on the availability of a link between a node and its nearest neighbour node.

Most importantly, an available link must also be reliable for a good quality communication to be established between node pairs. One major characteristic of the wireless channel that affects the quality of communication is the variation in its strength over time and frequency. As a result of the variation, communication links in wireless networks tend to be unpredictable. Moreover, this variation affects the connectivity between two communicating nodes.

Another factor that affects connectivity between two communicating nodes is mobility. Since mobility may cause connected radio links to be disconnected, a critical issue is for nodes in the network to be able to communicate on links that can last as long as the required packet transmission duration. Therefore the link between node-pairs has to be strong enough to ensure a lasting connectivity.

The developments of the theory of connectivity in wireless networks have been done in research works such as [1-7]. However, most of the theoretical and analytical investigations have been developed with ad-hoc and sensor networks in mind. This research studies the theory of connectivity in inter-working multi-hop wireless networks. The main contribution of this research is to provide an analysis of connectivity between any node pair in inter-working multi-hop wireless networks. Specifically, this research focuses on link and route availability between node pairs in inter-working multi-hop wireless networks.

In a wireless network, link and route availability depends on the distance between two node pairs and the transmission range of the nodes. Link and route availability are based on probability since the wireless network is stochastic in nature. The probability that a route is available between any source-destination node pair depends on the probability that all nodes on the route between the source and destination have a link to another node closer to the destination node. Link availability is the probability that two node pairs are within at most the maximum transmission range that is sufficient for a communication link to be established between them. Route availability is the probability that a certain number of links are available to form the communication route between source-destination pairs.

In particular, this paper presents the analysis of the relationship that exists between link availability and route

availability in inter-working wireless multi-hop networks. Such analysis is needed in order to establish the probability of route availability for source-destination node pairs. In this paper, a link refers to the connection between any node pair in the network, while a route refers to the last mile connection path between a source and destination pair.

In section II, a description of the model of the inter-working wireless network and the node distribution model; and an explanation of the node degree concept are presented. The analysis of link and route availability is given in Section III, while section IV concludes the paper.
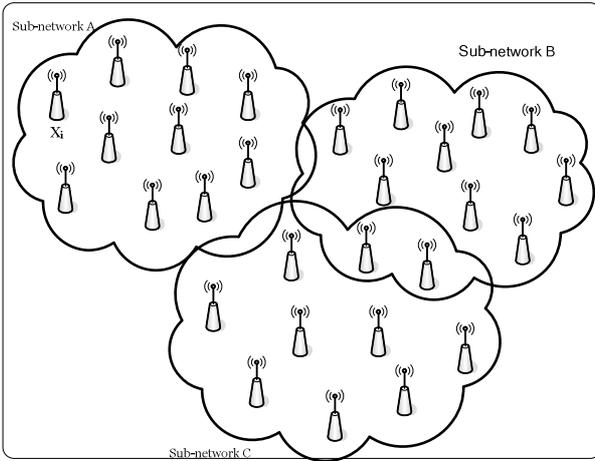
II. NETWORK MODEL

Figure 1.   Network Ω

The network (Ω) in fig 1 represents an inter-working wireless network. Network Ω contain three subset networks (sub-networks) A, B, C. The total number of nodes in Ω is denoted by N_Ω, while the number of nodes in each of these sub-networks (A, B, C) are N_a, N_b and N_c respectively, where $N_a + N_b + N_c = N_\Omega$ and $N_a = N_b = N_c = N$ (i.e. the sub-networks contain the same number of nodes). All the nodes have the same transmission capability and packets are transmitted from the source node towards the destination node via a multi-hop path.

A. Node Distribution Model

In these sub-networks in fig. 1, nodes are independently and randomly placed on a 2-dimensional area A, where the node density $\rho = N/A$ (number of nodes per unit area). These nodes are distributed randomly within the area A of the sub-network. The maximum transmission range of each of the nodes is R and the distance between any two nodes, Xi and Xj $\forall\, i,j \in Z\ and\ i \neq j$ in the network is represented by d(Xi,Xj). In a wireless multi-hop network, two nodes are able to communicate with each other if d(Xi, Xj)≤R [2].

B. Node Degree

The degree of a node in a wireless multi-hop network is defined as the number of neighbor nodes that it has [8]. A node is said to be a neighbor node to another node if the distance between them is less than or equal to their maximum transmission range, which means that both nodes are able to link directly to each other. Therefore, a node's degree is the number of nodes within its transmission range.

The node degree of a node Xi is denoted by D(Xi). In an instance where for a node, D(.) = 0, the node is termed a "lone node". The existence of a "lone node" in a multi-hop wireless network is an undesirable condition. For a static multi-hop wireless network, this type of node is totally useless to the whole network in terms of connectivity. However in a mobile scenario, a lone node becomes useful as it moves into the transmission range of another node or when another node moves into the node's transmission range. The desirable condition for any multi-hop wireless network is to have D(.) for all nodes greater than zero i.e D(.) > 0. The probability that D(.) > 0 for any node pair is the same as the probability that a link is available for the node, and the probability density function is given by equation 1. R is the transmission range of the node and f(x) depends on the distribution of the nodes in the network.

$$P(D(.) > 0) = P(link\ availability) = \int\limits_{o}^{R} f(x)dx$$

(1)
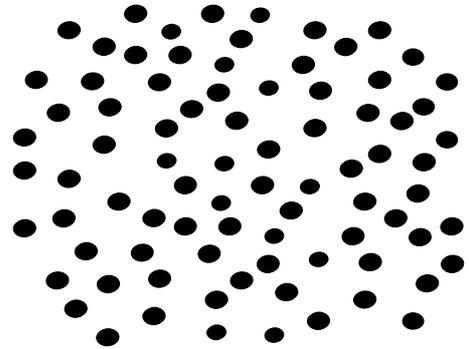
III. AVAILABILITY

Figure 2.   Spatial Point Pattern.

A. Link Availability

Consider each sub-network in fig.1 as a collection of random points (nodes or data or events) whose realization is called a spatial point pattern shown in fig.2 [9]. These nodes are contained in a Euclidean space of 2-dimensions (R^2), and their positions in the network are independent of each other. The lack of dependence between these nodes is called *complete spatial randomness (csr)* [9]. From theory, these nodes can be said to form a realization of a Planar Homogeneous Poisson Point Process. With regards to the

analysis of spatial point pattern, the distribution theory of such points (nodes in this case) under complete spatial randomness is well known under the theory of the Nearest Neighbor Distance (NND). These theories are used to analyze point patterns in biological sciences and are applicable to wireless networks [2].

Note that the distance of a node to its neighbor nodes is the nearest neighbor distance. Let β denote this distance, so that $\beta = d(Xi,Xj)$ $\forall i,j \in Z \ and \ i \neq j$. With theorem 1 given below, the probability that β is greater that R can be evaluated.

Theorem 1: *For a Homogeneous Poisson Point Process in* $\Re^2$ *(two dimensional plane), the probability that there are no point within a distance y of an arbitrary point (p) is* $e^{-\lambda \pi y^2}$, *where the parameter* λ *is the expected number of points per unit area [9, pg 636].*

The above theorem applies to any of the three sub-networks in fig. 1 in the following way:

1) For an arbitrary node in any of the three sub-networks, the probability that there are no nodes within a distance $\beta \leq R$, (probability that a node has no neighbor/probability that a node is a lone node) is:

$$P(\beta > R) = e^{-\rho \pi R^2} \ for \ R > 0 \tag{2}$$

where ρ is the number of nodes per unit area of each of the sub-network.

2) Also, for an arbitrary node in any of the three sub-networks, the probability that the distance between a randomly chosen node and any of its nearest neighbor node is less than or equal to the node's transmission range R (the probability that a node has at least one neighbor) is:

$$P(\beta \leq R) = 1 - e^{-\rho \pi R^2} \ for \ R > 0 \tag{3}$$

The availability of a link for a node (P_{link}) is as given by equation 3. P_{link} exists as long as $\beta \leq R$. A node becomes a lone node (no link is available) once $\beta > R$, and the probability of this occurring is given in equation 2.

In a multi-hop networks, P_{link} is a monotonically increasing function as shown in fig. 3. Fig. 3 gives a plot of the P_{link} and the normalized transmission range, R. The case of a network of N=20 nodes in an area of 10 square unit has been considered. The effect of the increase in the transmission range of nodes was observed. As R increases, P_{link} also increases because more nodes become available for a 1-hop link.

At R=0.2, only 22.2% of the total nodes are available for a 1- hop link to any chosen node and 99.8% of nodes are available if R=1. All (100%) of the links are available at higher values of, which means that every node has a link to all other nodes in the network, This phenomenon indicates that the network is fully connected.
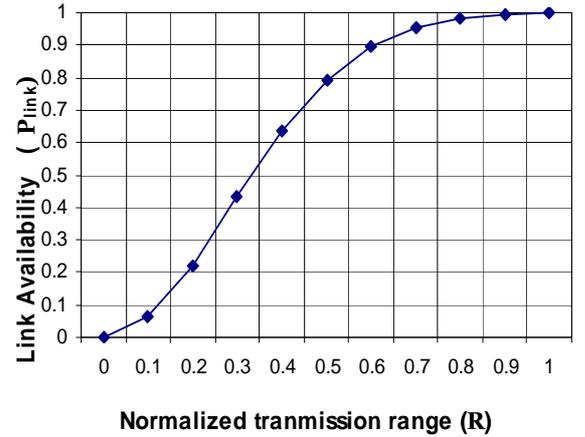
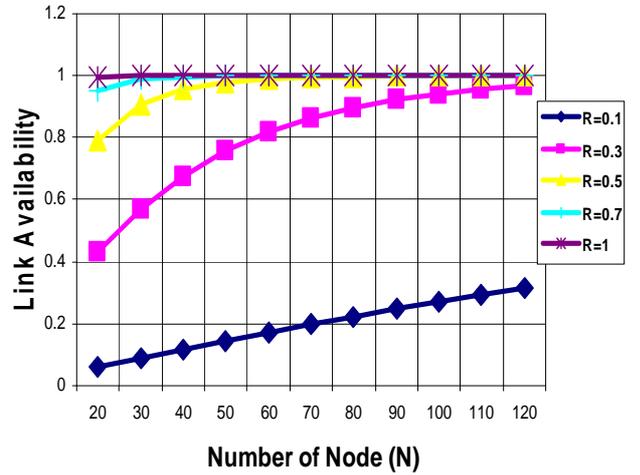

Figure 3. Link Availability vs Normalized R

Figure 4. Link Availability vs Number of Nodes (N) for different values of R.

Fig. 4 gives a plot of the link availability (P_{link}) at fixed transmission range (R) as the number of nodes in the network increases. The same area of 10 square units has been considered, but the number of nodes was increased from 20 nodes to 120 nodes at fixed values of R= (0.1, 0.3, 0.5, 0.7 and 1). Generally, P_{link} increases as the number of nodes increases, indicating that in multi-hop wireless networks, the probability of having a node-pair linked up is higher in a dense network. For high values of R, P_{link} is at very high values for a high node density network.

Then the upper bound for link availability between any node pair is given as:

$$P_{link-upper} = 1 - e^{-\rho \pi R^2} \ for \ R > 0 \qquad (4)$$

While the lower bound for the unavailability of a link is

$$P_{no-link-lower} = e^{-\rho \pi R^2} \ for \ R > 0 \qquad (5)$$

The availability of a link is a sufficient condition for connectivity, but it is not sufficient enough to ensure a reliable transmission of packets between node pairs. However, for simplicity at this stage, let's assume that the availability of a link (P_{link}) between any node-pair is dependent only on the distance between the nodes and that links in the network are identical.
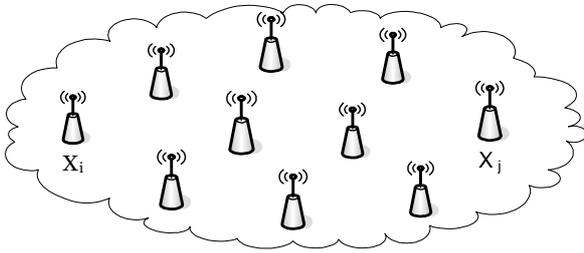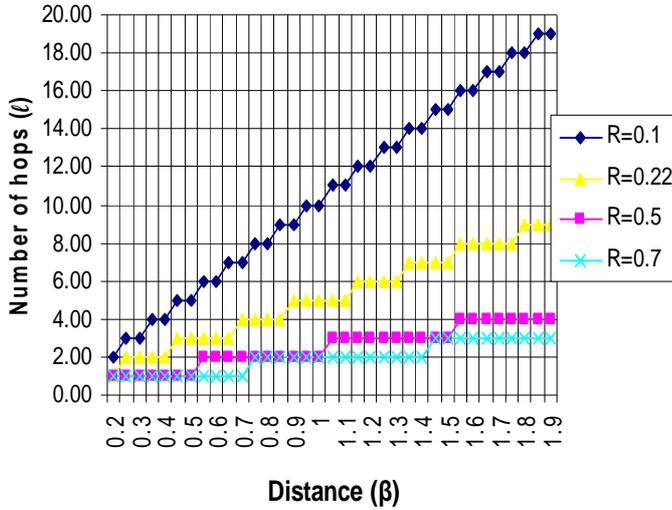
B. Route Availability

Figure 5. A sub-network

Figure 6. Number of hops (l) vs distance between node pairs

In inter-working multi-hop networks, it is unlikely that $\beta \le R$ will always be the case for all source-destination node pairs. In instances where $\beta > R$ multiple hops are utilized for communication. If $\beta > R$, it means that a link is not available between the source-destination node pairs. For this scenario, a route consisting of multiple links will be established between the nodes. The number of links (hops) that will be utilized

depends on the distance between the node pairs. The minimum number of links (hops) that can connect any two nodes together is given by:

$$l = \left\lceil \frac{\beta}{R} \right\rceil \qquad (6)$$

$\lceil x \rceil$ represents the greatest integer that is greater than x. For the same network scenario in section III A, fig. 6 confirms that the longer the distance between node pairs, relative to their transmission range, the more the number of hops (links).

Let ℓ represent the link between any two nodes in network Ω, where L is the set of all links that exists in the network. If a transmitted packet from a node have to hop on a total of ℓ links to arrive at the destination node, then ℓ-1 intermediate nodes will be required on this route. Note that in order to ensure end to end route availability, each intermediate node on the route must have at least two 1-hop neighbor nodes. These two neighbors are for the purpose of packet reception and transmission towards the destination node.

Consider a network with N nodes as in a sub-network in fig. 5. If a route is to be established between any two nodes Xi and Xj, where Xj is the specific target destination, then there are N-2 possible intermediate nodes between Xi and Xj. Depending on the value of β and R, the maximum number of hops that can link Xi and Xj in this network is N-1 and the minimum number of hop is 1.

If a route with a definite number of hops say ℓ hops is to be established between Xi and Xj, then only ℓ-1 nodes are required on this route, as stated earlier. Let G be the number of ways that intermediate nodes can be linked up to set up distinct routes of only ℓ hops between Xi and Xj. Since there are N-2 possible intermediate nodes for the connection and ℓ-1 nodes are required to establish an ℓ-hop route, then

$$G = \frac{(N-2)!}{((N-2)-(l-1))!} \ , 1 \le l \le N-1 \qquad (7)$$

From the equation 7, there are 8 distinct ways that a 2-hop route can be set up in a network with 10 nodes. For a network of N nodes, the probability of an ℓ-hop route ($P_{\ell-hop}$) between any source-destination pair, e.g. Xi and Xj, is given below for $1 \le \ell \le$ N-1,

$$P_{l-hop} = \frac{(N-2)!}{((N-2)-(l-1))!} (P_{link})^l$$

$$P_{l-hop} = G \ (P_{link})^l \qquad (8)$$

The derivation in equation 8 is based on the assumption that for all nodes in the network P_{link} exists and it's the same for all nodes. It is also possible to find $P_{\ell-hop}$ if the P_{link}

between any node pair in the network varies. For example, in the case where X_k (k takes values within Z such that $\forall i, j, k \in Z, i \neq j \neq k$) are intermediate nodes between X_i and X_j and $P(X_k,X_j)$ is the availability of a link from node X_k to node X_j . If $P(X_k,X_j)$ is the same for all intermediate nodes, X_k and the P_{link} for all other node pairs are also the same, then, $P_{\ell\text{-hop}}$ is given as in equation 9, for $1 \leq \ell \leq$ N-1.

$$P_{l-hop} = G(P_{link})^{l-1} P(X_k, X_j) \qquad (9)$$

In case of node or link failures, an alternative detour needs to be available at any point in order to ensure end-to-end packet transmission. This alternative route may require more than the minimum number of hops or the same number of hops as ℓ So now, what is the probability that in a multi-hop network, a source destination pair will be connected anyhow irrespective of the number of hops from source to destination? Let P_r denote the probability that a route is available.

$$P_r = \sum_{l=\frac{\beta}{R}}^{N-1} P_{l-hop} \qquad (10)$$

Equation 10 gives the route availability for $\beta/R \leq \ell \leq$ N-1. P_r, depends on the probability of establishing an ℓ-hop route, between any pair of source-destination node in the inter-working wireless network. It also depends on the node density and the transmission range of nodes in the network.

IV. CONCLUSION AND FUTURE WORK

In this paper an analysis of the link and route availability in wireless multi-hop networks has been presented. The research work focuses on route connectivity in inter-working multi-hop wireless networks.

In multi-hop wireless networks such as mobile ad-hoc networks, a network connectivity analysis is needed. However, in multi-hop inter-working wireless networks, an analysis of the route connectivity is more desirable. For there to be connectivity between a source-destination node pair in an inter-working multi-hop wireless network, a route has to be available.

Route availability is dependent on the availability of a link between the node pairs. A distance-dependent model of link availability has been assumed. However, this model does not accurately represent the stochastic nature of the wireless channel. For optimal resource dimensioning and quality of service in inter-working multi-hop wireless networks, the randomness in the wireless environment needs to be considered. The second part of this research work includes the physical layer factors that affect the availability of a link in the evaluation of P_{link}.

Although, in mobile multi-hop network, β would be a stochastic parameter, yet the channel model does not include

the effect of attenuation, interference and fading on the wireless channel. In the second part of this research work, the parameters that induce randomness into the wireless channel are considered in the development of a link reliability model for inter-working multi-hop wireless networks.

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