Approximate Discrete Time Analysis of the Hybrid Token-CDMA MAC System

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Abstract—In this paper a hybrid Token-CDMA based medium access control (MAC) protocol is considered. The MAC scheme is analytically modeled as a multiserver multiqueue (MSMQ) system in the case of a gated service discipline. We present an approximated discrete time analysis for the queue inside the system where the analysis is concerned with the general case in which the system accommodates several traffic classes. Each queue in the system is assumed to incorporate the input model, the vacation model and the buffer model. The packet arrival process is assumed to be a Poisson distribution, with the same rate for the queues in each traffic class, and data rate quality of service (QoS) is incorporated to regulate the input. Packet service time is modeled with independent, identically distributed random variables with geometric distributions. Moments of the packet delay are derived using the probability generating function approach.

Index Terms—Hybrid MAC design, MSMQ system, Quality of service, Discrete time analysis, Queue length, Packet delay.

I. INTRODUCTION

Token passing medium access control protocols for Ad-hoc networks are gaining popularity in recent years as they have the potential of achieving high channel utilization than CSMA type schemes [1] and are capable of including QoS guarantees [2]. There exist a plethora of papers that proposed MAC schemes using token mechanism for Ad-hoc networks [1], [2], [3], [4], [5], and [6].

Using a similar approach as [2] and [6], a new hybrid Token-CDMA MAC protocol that incorporated a quality of service guarantee (QoS) is proposed by [7], in which the token-based scheme implements code division multiple access (CDMA) techniques to resolve packet collisions and incorporates QoS mechanisms to the Ad Hoc networks.

In analytical terms, the proposed hybrid Token Multi-code CDMA MAC scheme can be considered as a system that consists of multiple queues which are serviced by multiple servers, where this configuration is commonly denoted as multiserver multiqueue (MSMQ) system. There exist three packet transmission schemes for MSMQ systems [12]. For the proposed system, the scheme is adopted where a queue may only poll a single server during packet transmission is adopted. All other servers in the network arriving at the queue during transmission must be passed onto the next queue in the network. The proposed MAC scheme has the identical system characteristics as the MSMQ system. In this case, the storage capacity at each queue is assumed to be infinite and the queuing discipline is FIFO at each queue. The service discipline is gated at all queues. The polling order is given by servers visiting queues in a fixed index order. The maximum number of servers that can simultaneously attend a queue is one.

With the compact notation introduced in [12] for MSMQ systems, the system studied in this case can be denoted as a G/M/G/∞/{1} queue model. There exists an abundance of literature on the MSMQ networks, however, majority of it implements the 1-limited service discipline [8], [9], and [10] as its analytical model or the queue may be attended by multiple servers. In [12] and [15], the analysis was extended by presenting an approximate analytical results for the average server cycle and vacation times, as well as approximated closed-form expressions for the average packet waiting time under 1-limited, gated and exhaustive service disciplines.

There exist only a few papers that discuss the same packet transmission protocol for the gated type service discipline and vacation system. Based on [12], [16] presented an approximated analysis for the proposed gated multiple-vacation queue that supports multiple traffic classes. Using the approximated approach, the queue vacation time was derived. As the DiffServ [18] is an emerging architecture for future IP- Networks, the proposed MAC scheme is designed to be adequately integrating multiple types of traffic, which is in term making provisions for future integrations.

The analysis generally follows that of [14], but will depart from it in the considerations of vacation distribution and the incorporation of data rate QoS guarantee. The current contribution investigates the proposed gated MAC system in discrete time. The proposed multiple-vacation queuing model is used to conduct the approximated discrete time analysis and derive the moments of queue length and packet delay using the probability generating function approach. The model considered is the extension of the results from [16] both regarding packet arrival and vacation period.

The remainder of this paper is organized as follows. Section II describes the analytical model in detail. The approximated discrete time analysis for packet departure process is described in section III. Section IV presents the approximated discrete time analysis on moments of the queue length and packet delay. Numerical results for simulation and analysis are presented in section V and conclusion is drawn in section VI.
II. MODEL DESCRIPTION

In the proposed hybrid MAC scheme [7], a token is circulating in the system distributing CDMA codes to queues. To access the channel, queues require capturing the code. This model is similar to the multiple-server model, where the queues need to be polled by a server to be granted access to the channel. CDMA codes in this case are considered as servers. Using this approach, the proposed analytical model is built based on a multiple-server scheme.

![System model](image1)

Fig. 1. System model

The system model consists of $M$ codes/servers $S_1, ..., S_M$ and $N$ queues $Q_1, ..., Q_N$, as shown in Fig. 1. The queue model in the system is displayed in Fig. 2. It is assumed that each server represents a code channel, each queue resembles a node and the system is operated in stable state. Each queue, $i$, is assumed to have an infinite capacity, into which packets arrive according to a Poisson process. The queues are categorized into $r$ different classes where the number of queues in each class is denoted as $q_1, q_2, q_3, \ldots, q_r$. Queues in each class have the identical mean packet length is assumed to be geometrically distributed with a mean of $s$ bits/packet. Since the code channel rate is the mean of $s$, the packet length is assumed to be geometrically distributed with a mean of $\mu^{-1}$ slots/packet.

To provide data rate QoS, a modified leaky-bucket input regulation system is implemented. Each queue has a permit pool for storing the generated permits. The permit generation rate $1/T_p^{\text{gen}}$ is proportional to the data rate $T_p^{\text{gen}} = (\rho, \lambda_{ip}, \gamma_i)^{-1}$; $\rho_i \geq 1$. The packets that arrived have to gain the permission through leaky-bucket policing mechanism, where it must obtain a permit from a permit pool. The permit pool has a maximum capacity of up to $\gamma_i$ permits. If the generated permit finds that the pool is full, it is discarded. The packet length is assumed to be geometrically distributed with the mean of $s$ bits/packet. Since the code channel rate is assumed to be constant in the analysis, the service time of a packet is then also geometrically distributed with a mean of $\mu^{-1}$ slots/packet.

As mentioned earlier, when the token visits a queue and there is a code available, the gated-service discipline is employed where the server will empty the packet buffer $Q_j$ as shown in Fig. 2, detailed description of the transferring of packets between queues is discussed in section IV. After servicing the packets, the queue returns the code to the token and enters the vacation period. The vacation ends when the token with codes visits the queue again. The approximated mean value analysis of the vacation period is presented in [16]. The analytical model is assumed to be symmetrical. In this case it is assumed that all servers are identical and carry the same load.

![Queue model](image2)

Fig. 2. Queue model

III. DISCRETE TIME ANALYSIS FOR PACKET DEPARTURE PROCESS

The queue model in the system consists of three buffers ($Q_1, Q_2$ and $Q_3$) as shown in Fig. 2. The analysis starts with the queue input model where in this case the probability generating function of the packet departing process from $Q_1$ to $Q_2$ is derived. Under the discrete time scenario, the time is slotted where the length of the each slot $n$ is the permit generation slots with fixed slot length $T_p^{\text{gen}}$ as depicted in Fig. 3.

![State diagram](image3)

Fig. 3. State diagram of the discrete time system

From the queue model displayed in Fig. 2, channel access of the packets can be clearly described. In order to satisfy the proposed data rate quality-of-service (QoS) guarantee, packets arrive into an infinite buffer ($Q_1$ in Fig. 2) according to a Poisson process with mean arrival rate $\lambda_{ip,j}$ for queue $i$.

An arriving packet that finds the permit pool nonempty, departs the packet buffer to $Q_2$ and one permit is removed from permit pool. An arriving packet that finds the permit pool empty joins the buffer $Q_1$. When the queue is not empty and a permit is generated, one packet departs the buffer $Q_1$ immediately (in FIFO order) and the permit is removed from the pool. It can be observed that the packet departure process from this modified Leaky-Bucket scheme constitutes the input process to the network that is intended to be regulated.

For the analysis, the probability mass function (pmf) of the departing packets after the packet goes through the Leaky-Bucket ($P_{bh}$) at the slotted time is first derived before one can derive the pgf for the process. It is known that the amounts of packets departing the leaky-bucket during the $n$th slot are dependent on the queue length prior to permit
generation instances [17]. For the probability distribution of the
buffer length of \( Q_i \), its pdf can be found at embedded permit
generation point where the input queue can be modeled as an
M/D/1 queue [13]. For the M/D/1 queue, its queue length
distribution in steady state can be derived as,

\[
P_{q_i}(a) = \Pr[q_i = a] = (1 - \rho) \sum_{i=1}^{a} (-1)^{i-1} \rho^{i-1} \frac{(k \rho)^{a-i}}{(a-k-1)!} \left( \frac{(k \rho)^{i-1}}{(a-k-1)!} \right)
\]

where \( \rho = \frac{\lambda}{\mu} \) and with initial conditions,

\[
P_{q_i}(0) = (1 - \rho) \quad \text{and} \quad P_{q_i}(1) = (1 - \rho)(e^{\rho} - 1)
\]

(2)

The number of packets that arrived to \( Q_2 \) in \( nth \) slot
is dependent on the number of packets depart from \( Q_1 \) during
the \( nth \) slot. At the beginning of the \( nth \) slot, there may be residual
permits left from the previous slot, therefore the number of
packets that can depart from \( Q_1 \), during the \( nth \) slot is \( B_n \)
\( (0 \leq B_n \leq m + L_i) \), where \( L_i \) is the permit pool capacity for queue
model \( i \). Using the iterative process, the probability mass
function for the departing packets can be derived.

Using the iterative process and memoryless characteristic of
the M/D/1 queue, the steady state probability distribution of the
departing packet at slot boundary can be clearly derived as,

\[
P_{q_i}(0) = \Pr(q_i = 0) = (1 - \rho) \\
P_{q_i}(n) = \Pr(q_i = n) = \rho \cdot \sum_{j=0}^{n} \Pr(q_{i-1} = j) \\
\]

(3)

And for one packet onwards,

\[
P_{q_i}(n) = \Pr(q_i = n) = \rho \cdot \sum_{j=0}^{n} \Pr(q_{i-1} = j) \\
\]

(4)

It is known that in order to have more than one departure
within the \( nth \) slot, there must be residual permits from
previous slot and the maximum number of departures is
dependent on the permit pool size \( L_i \). In this case, there can only
have two departures in \( nth \) slot when there is one residual
permit from previous \( (n-1)th \) slot. Therefore the probability of
having two departures \( (P_{q_i}(2)) \) is equal to the probability of
having two packets in queue and with one residual permit from
previous slot \( (P_{q_i}(1)) \), which is illustrated in (3). The term
\( P_{RP}(b) \) is defined as the probability of having \( b \) residual
permits and it is determined by the queue capacity from the previous
sub slot. There will only be a residual permit available only if
the queue length from the previous slot is zero, therefore the
probability can be derived as,

\[
\text{Prob(Residual Permit } = b) = P(RP = b) \\
= P_{RP}(b) \\
= P(Q = 0 \text{ at previous slot}] \cdot P(Q = 0 \text{ at previous 2 slots})...
\]

(5)

\[
= P_{q_{i-1}}(0) \cdot P_{q_{i-2}}(0) \cdot \ldots \cdot P_{q_{i-b}}(0) \\
= \left( P_{q_i}(0) \right)^b
\]

(3) and (4) can simplified to,

\[
\Pr(B_n = j) = \begin{cases} 
(1 - \rho) & , j = 0 \\
(\rho)^j & , j = 1 \\
(1 - \sum_{j=0}^{n} \Pr(q_i = j)}(1 - \rho)^j & , 2 \leq j \leq L_i + 1
\end{cases}
\]

(6)

The probability generating function of the packet departing
process \( B(z) \) can now be derived using standard z-transform
method,

\[
B(z) = \sum_{j=0}^{\infty} \Pr(B_n = j) \cdot z^j
\]

(7)

\[
= (1 - \rho) + (\rho)^2 \cdot z + \sum_{j=0}^{L_i} \left( 1 - \sum_{j=0}^{n} \Pr(q_i = j) \right) (1 - \rho)^j \cdot z^j
\]

IV. DISCRETE TIME ANALYSIS FOR MOMENTS OF QUEUE
LENGTH AND PACKET DELAY

For transferring packets between queue buffers, packets that
depart \( Q_i \) arrive at \( Q_2 \) and wait in the queue before the gate and
move in batch to the queue \( Q_2 \) only when the gate opens. The
gate is only opened at the end of the last slot of the vacation
period. Once the gate is opened the packets in \( Q_2 \) are then
transferred to \( Q_j \) and are then served according to FIFO
principle before departing the system. A vacation starts when
\( Q_j \) empties and the gate opens at the end of each vacation.
However, if the server finds \( Q_j \) empty upon returning from
vacation, it will immediately start another vacation until \( Q_j \) has
packets when the server returns from the vacation (multiple
vacation policy). The mean value of the vacation length is
derived from [16] and it is modeled as a Polya distributed
random variable with probability mass function \( \rho \),
and corresponding probability generating function \( F(z) \).

For the queue model under consideration, a queue cycle is
defined to consist of a busy period that follows with a vacation
period. When the busy period starts, the queue content in \( Q_i \) is
emptied by serving all the packets , all the packets that arrived
during the busy period is stored in \( Q_i \) since the gate is closed in
busy period. Once the server has served the last packet in the
\( Q_i \) is then transferred to \( Q_j \) and the server moves to the next queue in the system. In this case the
gate takes on the vacation after it finished serving the packets
in \( Q_j \) is considered. At the end of the vacation period, the gate is
opened and all the packets that stored in \( Q_j \) are now conveyed
to \( Q_j \) in the FIFO order.

\( c_{l+1} \) is defined as the lth cycle and let \( X_l \)
as the number of packets in the \( Q_j \) at the beginning of the slot \( i \).
[14] indicated that the number of packets in the \( Q_j \) at \( c_{l+1} \) can
then be defined as,
where \( g_i \) is defined as the service time of the packet \( i \) during the \( l \)th cycle, \( B_i^j \) is defined as the number of departures from \( Q_j \) to \( Q_j \) during the \( j \)th service slot of the packet and \( W_{l+1} \) is defined as the number of departures from \( Q_j \) to \( Q_j \) during the vacation period in the \( l+1 \)th cycle. \( X_{o_l}(z) \) is defined as the probability generating function of the number of packets in the queue at the end of the \( l \)th cycle; its pgf can be shown as [14],

\[
X_{o_l}(z) = X_{o_0} \left( S(B(z)) \right) \overline{W_0}(z) + X_{o_0} \left( S(b_0) \right) \left( W_0(z) - \overline{W_0}(z) \right)
\] (9)

where \( W_0(z) \) is defined as the probability generating function of the number of departures from \( Q_j \) to \( Q_j \) during the vacation period of a random cycle under the condition that there exists no packets in \( Q_j \) at the end of the slot preceding the vacation period. \( \overline{W_0}(z) \) is defined as the probability generating function of the number of departures from \( Q_j \) to \( Q_j \) during the vacation period of a random cycle under the condition that there is minimum one packet in \( Q_j \) at the end of the slot preceding the vacation period. It can be easily derived that \( \overline{W_0}(z) = \nu(B(z)) \) since under the condition that if there are at least one packets in \( Q_j \) before the vacation starts, the server will then only take one vacation. However, the server will take multiple vacations until there exists a packet in the \( Q_j \) when it comes back from the vacation. Modifying the analysis from [14] and by conditioning on the number of necessary vacations,

\[
W_0(z) = \frac{\nu(B(z)) - \nu(b_0)}{1 - \nu(b_0)}
\] (10)

To find the probability generating function of the queue length at the end of the cycle, \( X_c(z) = \lim_{l \to \infty} X_{o_l}(z) \) is defined as its pgf for the queue length at the end of the cycle in equilibrium. It is proven from [11] that this condition is valid under the assumption where,

\[
\delta_i = S_i(1)B_i(1) < 1
\] (11)

where \( \delta_i \) is the load of the queue model \( i \) and from the equilibrium assumption, (9) can now be derived as,

\[
X_c(z) = X_{o_0} \left( S(B(z)) \right) \overline{W_0}(z) + J \left( W_0(z) - \overline{W_0}(z) \right)
\] (12)

where \( J = X_{o_0}(S(b_0)) \) is the probability that \( Q_j \) is empty before the start of the vacation period. It is now clearly shown that various moments of \( X_c(z) \) can be derived using implicit determination and that the value of \( J \) can be determined numerically using recursive technique [14].

For the queue length at the end of packet service, \( X_d(z_1,z_2) \) is defined as the joint probability generating function of the queue length at the \( Q_j \) and \( Q_j \) at the start of the slot right after a service of a packet from \( Q_j \), its pgf can then be derived from [14] as,

\[
X_d(z_1,z_2) = \frac{X_c(z_1)S(B(z_2)) - X_d(z_1)S(B(z_2)) - X_c(z_1)z_1}{S(B(z_2)) - z_1}
\] (13)

Where \( X_{d,1} \) and \( X_{d,2} \) are defined as the queue length in the \( Q_j \) and \( Q_j \) at a random service epoch respectively and \( X_c \) is previously defined as the total queue length in \( Q_j \) and \( Q_j \) at the end of the random cycle.

In the discrete time analysis, the packet delay is denoted as the number of slots between the end of the slot the tagged packet arrives in at \( Q_j \) and the end of the slot where that tagged packet leaves \( Q_j \). The service time of the packet is taken into consideration in determining the delay from \( Q_j \) as the packet only departs from the queue once it is being served. For the modified leaky-bucket QoS scheme, the exact delay expression for packets in \( Q_j \) has been derived by [13] therefore this paper concentrates on the delay expressions on \( Q_j \) and \( Q_j \). The number of packets that depart from \( Q_j \) in a slot are grouped to form a “batch customer” which forms a system with Bernoulli “batch-customer” arrivals. The probability generating function for the departures \( B'(z) \) and their service times \( S'(z) \) are given by,

\[
B'(z) = b_0 + (1 - b_0)z,
\]

\[
S'(z) = \frac{B(S(z)) - b_0}{1 - b_0},
\] (14)

Let \( X_d'(z_1,z_2) \) denote the probability generating function of the \( Q_j \) and \( Q_j \) queue length at departure epochs for this system. By considering a random batch packet and let \( D'(z_1,z_2) \) denote the joint probability generating function of its delay in \( Q_j \) and \( Q_j \) queues. For the batch-packets that arrive during its delay in the \( Q_j \) are moved to \( Q_j \) along with the tagged batch-packet. And for all the batch-packets that arrive during its departure in the \( Q_j \) are present in the \( Q_j \) at its departure. It is then shown by [14] that the probability generating functions of batch-packet delay and queue contents at batch-packet departure epochs can be easily related as,

\[
D' \left( B'(z_2), S'(z_1) \right) = X_d'(z_1,z_2)
\] (15)

To relate the delay of the packet to the delay of its batch, the delay in \( Q_j \) of a packet equals the delay of its batch as they enter and leave \( Q_j \) at the same time slot. The waiting time of a packet is denoted as the number of slots between the end of its arrival slot and the beginning of the slot where this packet starts its service. Therefore the waiting time in \( Q_j \) of a packet is then the sum of the waiting time of its batch, with combination of the service times of all packets that arrived during the same slot prior to the tagged packet. With modification from [14], the packet delay in \( Q_j \) and \( Q_j \) can be shown as,

\[
D(z_1,z_2) = \frac{S(z_1)}{B'(1)S'(z_1)} \left( b_0 + \frac{z_1 - b_0}{1 - b_0} \right) \left( \frac{b_0 + \frac{z_1 - b_0}{1 - b_0}}{1 - b_0} \right)
\] (16)

Various moments of the packet delay for both \( Q_j \) and \( Q_j \) queue buffers can now be derived using derivatives techniques for (16).

V. NUMERICAL RESULTS FOR MEAN PACKET DELAY AT \( Q_j \) AND \( Q_j \) QUEUE BUFFERS
In this section numerical results are presented to illustrate the computation results from the proposed analytical MSMQ model for the hybrid MAC scheme with comparison to the simulation. Simulation and analysis are conducted using the parameters shown in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
</tr>
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<tbody>
<tr>
<td>N</td>
<td>9</td>
</tr>
<tr>
<td>M</td>
<td>6</td>
</tr>
<tr>
<td>r</td>
<td>3</td>
</tr>
<tr>
<td>$q_i$</td>
<td>3</td>
</tr>
<tr>
<td>$\lambda_i^1$</td>
<td>$1.5\lambda_i^3$</td>
</tr>
<tr>
<td>$\lambda_i^2$</td>
<td>$2\lambda_i^3$</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>1.2</td>
</tr>
<tr>
<td>$\mu^{-1}$</td>
<td>1.25 sub-slot/packet</td>
</tr>
<tr>
<td>$h$</td>
<td>0.01 sub-slot</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 4, 5 and 6 also show that the analysis result of the delay for all classes under heavy system load conditions compare favorable to simulation results. However, the effect of the independence assumption on the vacation time distribution starts to dominate which leads to deviation between analysis and simulation. For the mean packet delay in queue buffer $Q_3$ of different class queues, the simulation and analysis results for various system load conditions are displayed in Fig. 7, 8 and 9. Clearly, when the system is under a light to medium traffic condition, the probability that a packet arrives during the cycle is low, therefore leads to low packet delay for all classes.

Fig. 4, 5 and 6 display the results of the mean packet delay experienced by buffer $Q_2$ for all traffic class queues. From the figures, class 3 queue buffer has the highest delay amongst all the class queue buffers and class 2 buffer has the lowest delay. This is expected as for the system under consideration, class 3 queue has the highest data rate therefore more packets are stored in the queue comparing to other classes subsequently leads to the increase in packet delay.

VI. CONCLUSION

The paper presented the analytical model of the hybrid Token-CDMA MAC scheme with gated service discipline and
data rate QoS. Approximated discrete time analysis was also conducted for the packet departure and for the moments of packet delay at various queue buffers. Some numerical examples for the proposed analysis are presented, and it was illustrated that the analytic results compare favorably to simulation results. However, in the low utilization range, the effect of the independence assumption dominates which leads to deviation between the results.

REFERENCES


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