# Components of electrically small loop antennae:1

In the first part of this series, B A Bowles, B A Austin and C W P Attwell outline the fundamentals of underground radio transmission with particular emphasis on loop antennae

Electrically small loop antennae are being used increasingly for special applications in the HF and MF bands for the transmission and reception of radio signals. Their use has been necessitated by the requirement for small, low profile and in many cases portable antenna systems. In particular, in marine and military applications the loop antenna is a smaller and hence more suitable structure than larger ship or rod antennae which produce an equivalent performance.

In the South African underground gold mines small, multi-turn loop antennae operating in the frequency range 100 kHz to 1 MHz have been in daily use for over three years. These mines are characterised by extremely rugged conditions with mining areas (stopes) frequently less than a metre high.

The ability to propagate radio energy successfully through rock has led to other interesting applications such as radio communication in buildings which is of interest to fire fighting and security personnel.

The design of electrically small loop antennae has been the subject

The authors are with the Chamber of Mines of

of much investigation. However, for the antenna to operate effectively a matching network is required to overcome the large reactive component of the antenna input impedance and thereby present conjugate impedances and therefore maximum power transfer between the antenna and the external circuitry.

The small "electrical" size of the antenna contributes to an inherently low radiation efficiency and care must be taken with the selection of all matching components to optimise the performance. The design of the matching network must take into consideration the system bandwidth, the phase response, and the impedance it presents to the transmitter and to the receiving circuits.

Unfortunately, although various matching networks have been suggested and many used, there has been no previous mention on the influence of the matching network on the system design.

Part 1 of this article outlines briefly some fundamentals of underground radio propagation and describes an equivalent circuit for the electrically small loop antenna in the frequency range 100 kHz to 3 MHz. The loss caused by the equivalent components of the antenna and the components of the matching network are described and practical results given. In Part 2, the equivalent circuit is used to derive the theoretical frequency responses of the important parameters of various matching networks whose properties are then compared.

# Loop antenna

An antenna worn across the chest and over one shoulder forms a more convenient mechanical configuration than a rod or whip antenna when used in the confined low stoping widths of underground mining areas. Furthermore, when surrounded by a lossy medium, a magnetic dipole (small loop) possesses better radiation characteristics than an electric dipole, (rod or whip antenna) since the power radiated by a loop is dissipated less rapidly with distance from the source in the surrounding rock than that radiated by the rod antenna.

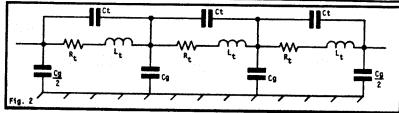
Loop antennae surrounding a high permeability core, such as ferrite are in common use as receiving antennae

Fig. 1: System loss through quartzite Fig. 2: Equivalent circuit

Table 1: Performance of various capacitor types compared

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System loss in quartzite-d8	230-	1 1111	ı
	220-	/	١
	210-	300m	1
	200-	/ 1111	١
	190-		1
	180-	<b>→</b> ////	1
	170-	2/11	
	160-		
	150-	1200	١
	140-	900m	١
	130-	600m	١
	120-	300m	۱
	110-	* /	П
	100	335kHz	
	30	100 300 1000 3000 100	8
Fig	j. 1	Frequency kHz	

Capacitance type	Resonant frequency (kHz)	Capacitance value (μF)	Total loss resistance (Ω)
Polyester	584	3.64	1.51
Polycarbonate	635	2.98	1.05
Polystyrene	601	3.29	0.55
Silver mica	629	3.0	0.53
Polyester	202	21.8	0.58
Polycarbonate	237	21.3	0.34
Polystyrene	248	19.5	0.25
Polypropylene	231	22.4	0.25
Silver mica	233	21.8	0.23



in the MF broadcast band and are used in certain mine communication equipment. However, in portable applications, ferrite antennae are restricted in size, which limits their power handling capability due to core saturation.

Furthermore, when mounted in the transceiver they are subject to movements of the transceiver and hence the radiation field pattern is not as stable as that when the loop is worn over the shoulder. This is particularly important in the cramped conditions of mining areas and since our experience has shown that, for direct propagation through rock, ferrites have produced no greater values of field strength, loop antennae are preferred for their practical advantages.

# **Basic considerations**

One of the objectives of antenna design is to achieve the maximum signal at the receiver for a given transmitter power. In many situations the antenna is in the far field of the transmitter antenna and in free space the magnitude of the electrical field component at a distance r and at an angle of  $\theta$  to the normal axis of an electrically small loop antenna with n turns is given by:—

$$E_{o} = \frac{120\pi^{3} \text{ I } \sin\theta \text{ a}^{2} \text{n (V.m}^{-1})}{\text{r}\lambda^{2}}$$

Where I = antenna current(A)

a = radius (m)

 $\lambda = \text{signal wavelength (m)}$ 

In underground applications, the actual magnitude of the far field is influenced by the electrical characteristics of the rock, the effect of the rock on the antenna characteristics and by the presence of indirect coupling into metallic conductors such as traction lines and electrical distribution systems. However, the input voltage to the receiver still

depends on the transmitter antenna current and it is necessary to design the antenna and transmitter in order to achieve maximum circulating current subject to the system bandwidth.

Optimising the system between the transmitter and receiver circuits presents considerable problems since the interaction between the antenna and the surrounding lossy medium is still not fully understood. Earlier studies only considered the antenna as a free space element but it has been shown that the characteristics of an underground antenna are dependent on the transmission medium. Consequently it is still not possible to define an optimum frequency for communication but it is known to be less than 3 MHz, since the rock attenuation increases considerably for higher frequencies.

Allowing for the previous limitations a rough insight into the total system loss can be gained by adding the free space antenna loss to the rock attenuation to give the set of curves in Fig. 1. These curves indicate that for a given range there is an optimum frequency of operation; for a typical distance of 900 m this frequency is about 300 kHz. For most cases the frequency of operation is between 100 kHz and 1 MHz.

### **Equivalent circuit**

This section describes the components of a loop antenna, together with the effects of the matching network and the surrounding medium.

The equivalent circuit of an electrically small loop antenna is shown in Fig. 2. Each turn of the antenna is represented by an inductance (L<sub>1</sub>) in series with a resistance (R<sub>1</sub>) which is the sum of the loss resistance and the radiation resistance. Also present is a distributed inter-turn capacitance

 $(C_t)$  and a capacitance to ground  $(C_g)$ .

In order to design and understand the operation of an antenna and matching network system it is necessary to have some idea of the magnitude of the various equivalent components and the effect of these components on various measured parameters.

The low frequency inductance of an isolated low loss multi-turn (n) loop antenna can generally be found from a formula of the form:

 $L = Cn^2$  (Henries)

The constant C depends on the dimensions of the antenna and has different values for various configurations. When measuring the inductance, care should be taken with the choice of the measurement frequency (f<sub>m</sub>) since the apparent value of inductance changes as the antenna self-resonant frequency is approached.

If the antenna capacitance to ground is ignored for the present, then the circuit for frequencies below resonance is equivalent to an effective inductance (L<sub>e</sub>) in series with an effective loss resistance (R<sub>e</sub>). The magnitude of L<sub>e</sub> and R<sub>e</sub> are related to the total inductance (L), total loss (R) and total inter-turn capacitance (C) by:—

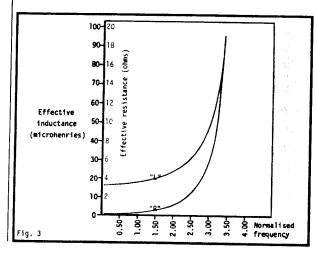
$$L_{e} = \frac{L(1 - \omega^{2}LC) - CR^{2}}{(1 - \omega^{2}LC)^{2} + (\omega CR)^{2}}$$

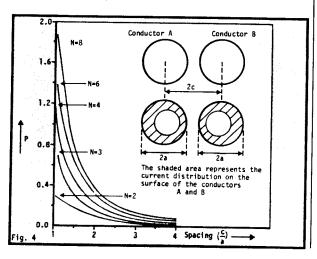
$$R_{e} = \frac{R}{(1 - \omega^{2}LC)^{2} + (\omega CR)^{2}}$$
(ohms) (3)

where  $\omega = 2\pi f_m$  (rad. sec<sup>-1</sup>) The variation of L<sub>e</sub> and F

The variation of L<sub>e</sub> and R<sub>e</sub> with frequency for a typical loop antenna constructed from four-core mains

Fig. 3: Effective inductance/resistance
Fig. 4: Variation of P with number of
conductors





cable and which has four turns, a diameter of 45 cm, a low frequency inductance of 21  $\mu$ H and a stray capacitance of 91 pF, is shown in Fig. 3.

At a frequency of 913 kHz, which is approximately one quarter of the self-resonant frequency, the antenna factor  $\left(\frac{\omega L}{R}\right)$  is 107, the effective inductance is 1.07 L, the effective resistance is 1.14 R but the apparent

Q factor (R) is 6% lower.

At one half of the self-resonant frequency the apparent Q factor is reduced by 30%. However when the antenna is connected to a current source the circulating current in the inductance will behave as if the antenna has a Q factor of 107 and hence the antenna bandwidth will be smaller than that expected from measurements at the operating frequency.

The antenna inductance is also influenced by the presence of conducting objects. This is particularly important in mining applications since the surrounding rock is a lossy medium and frequently use is made of conducting objects such as power cables and traction lines to enhance radio propagation. The effect of the rock is to increase the antenna impedance and it has been shown that the increase of impedance ( $\Delta Z$ ) with frequency (f) when situated at the centre of a hollow sphere of permeability  $\mu$  and conductivity  $\sigma$  is given by:

r

١t

t

d

8

$$\Delta Z = \frac{(2\pi f\mu)^2 \sigma \pi^3 b^4}{6a} X$$

$$\left\{ 1 + \frac{1}{15} \left( \frac{b}{a} \right)^2 + \frac{1}{70} \left( \frac{b}{a} \right) \right\} (4)$$

Fig. 5: Variation of tan 8 with frequency Fig. 6: Effect of frequency on antenna loss

where b = the antenna radius (metres) and a = the sphere radius (metres).

In a typical situation,  $\mu = 4\pi \times 10^{-7} \text{ H.m.}^{-1}$ ;  $\sigma = 2 \times 10^{-5} \text{ S.m}^{-1}$ ; a = 1 m; b = 25 cm; f = 335 kHz, then  $\Delta Z = 3 \times 10^{-7} \Omega$ .

Hence for the majority of mining applications the surrounding rock has very little effect on the antenna impedance. However, significant changes occur due to the presence of metallic objects and in order to observe the effect the inductance of the antenna used for Fig. 3 was measured as it approached a 1 m square copper plate.

The values measured are as follows:

isolated antenna —  $21.5\mu H$ antenna 20 cm above object -  $20.9\mu H$ 

antenna 2 cm above object —  $15\mu H$ 

antenna 2 mm above object —  $8.5\mu H$ .

The decrease in inductance is attributed to the flux generated by the induced currents in the conducting object and the effect is similar to that observed when a coil is placed inside a conducting cylinder.

The effectiveness of an antenna as a radiator is often described by its radiation efficiency which is defined as the ratio of radiated power and total input power.

Radiation Efficiency =

$$\frac{I^2 R_r}{I^2 (R_r + R_1)} = 1 + (R_1/R_r)^{-1}$$
 (5)

where  $R_r$  = radiation resistance (ohms),  $R_l$  = antenna loss (ohms).

Hence the maximum radiation efficiency is obtained when  $R/R_r$  is a minimum. However, the design of an antenna system is complicated by the complexity of  $R_1$  since it involves the

following components:

$$R_1 = R_{dc} (1 + S) (1 + P) + R_m + R_c$$
 (ohms) (6)  
where  $R_{dc}$  = dc resistance;  $S$  = modification of  $R_{dc}$  by the skin effect;  $P$  = modification by the proximity effect;  $R_m$  = loss produced in the antenna matching network and  $R_c$  = the loss produced by coupling into surrounding objects.

In order to obtain a better understanding of the problem it is worth considering the relative magnitudes of various resistance terms.

# Radiation resistance

The free space radiation resistance of a (n) turn loop antenna having a diameter (2a) which is less when <sup>1</sup>/<sub>10</sub> wavelength is given by:

$$R_r = 31,200 \quad \left[ \begin{array}{c} -n\pi a^2 \\ \overline{\lambda 2} \end{array} \right]^2 \text{ ohms } (7)$$

Hence for a typical antenna with: n = 3, a = 25 cm,  $\lambda = 300$  m then  $R_r \approx 3 \times 10^{-6}$  ohms.

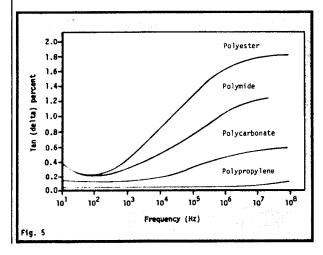
When the antenna is surrounded by a lossy medium it is difficult to compute a value for  $R_r$  since the medium affects the operation of the antenna. Furthermore, the standard methods of calculating  $R_r$  also do not apply due to the progressive dissipation of energy away from the antenna. However, it is expected that the radiation resistance is of the order of the above value,  $(3 \times 10^{-6} \text{ ohms})$ .

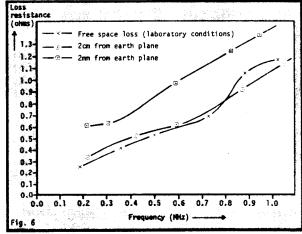
### DC resistance (R<sub>dc</sub>)

This term is evaluated easily and is typically  $0.1\Omega$  for the antenna described previously.

# Skin effect factor (S)

The skin effect on conductors is the result of eddy currents which





flow at high frequencies and thereby produce an ac resistance that is greater than the dc resistance. The ac resistance per cm of an isolated cylindrical copper conductor of diameter D (cm) at a frequency f (MHz) is given by:

$$R_{ac} = R_{dc} (1 + S) = \frac{83 \text{ f}}{10^{9} \text{D}}^{\frac{1}{9}} \text{ ohms (9)}$$

e.g. for f = 200 kHz, D = 0.5 mm, then  $R_{ac} = 1.5\Omega$  per 16 m.

# Proximity constant (P)

As the spacing between conductors is reduced the proximity effect causes an increase in the ac resistance which can be larger than that due to skin effect alone. The magnitude of the proximity effect for an electrically small loop antenna has been investigated by Smith', who obtained experimental results which correlate closely with theory. In particular he found that the additional resistance factor (P) is a function of the number of conductors and the spacing between them. The relationship he derived between these variables is shown in Fig. 4. From these results the value for P for a 4 conductor arrangement having c/a = 2 is

# Matching network loss (R<sub>m</sub>)

Either one or two capacitors are used in most matching networks to overcome the inductive reactance of the antenna. Resonant circuits are, therefore, formed and it is important that the capacitors chosen have a low series resistance and a low dissipation factor (tanδ) at the frequency of operation in order to avoid absorbing any significant power. For example at a frequency (f) a lossy capacitor (C<sub>p</sub>) may be represented by a capacitance (C<sub>s</sub>) in series with a loss resistance (R<sub>s</sub>) with:

$$R_r = \frac{\tan \delta}{2\pi f C_s}$$
 (ohms);

 $C_s = C_p (1 + \tan^2 \delta) (farads) (10)$ 

If  $tan\delta = 0.02$  (typical polyester at 200 kHz)

f = 200 kHz

 $C_p = 0.01 \mu F$ 

then  $R_s = 1.6\Omega$ 

The variation of tanδ with frequency for various dielectrics is shown in Fig. 5. Although not shown, silver mica and polystyrene have dissipation factors comparable to those of polypropylene and paper

capacitors have dissipation factors comparable to those of polyester.

Although polystyrene capacitors have a low loss their use is often prohibited by their large physical size and for some types by a comparatively large inductive component. For example, a typical 0.02 µF capacitor may have a series resonant frequency as low as 2 MHz.

Silver mica capacitors have a low temperature coefficient (50 ppm per °C) and excellent high-frequency long-term stability (0.01%) but become bulky and difficult to obtain for values greater than 0.05  $\mu$ F. Care should be taken with the selection of these capacitors since not all types are encapsulated to take full advantage of the inherent qualities of silver mica.

Furthermore, the current carrying capability of silver mica is limited to less than 100 mA for good long term stability and hence for transmitter applications they are not as good as

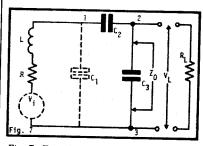


Fig. 7: Experimental matching network

the high frequency stacked-plate type.

For transmitter matching networks in the frequency range 100 kHz to 1 MHz, polypropylene capacitors may be used. These capacitors are not as stable (0.5%) as the best silver mica capacitors but experience has shown that they have comparable losses and they have been used successfully in many matching networks.

In order to compare the performance of various capacitor types for matching network applications a few specimens were inserted into a series resonant circuit and the input resistance was calculated at resonance. The resistance was found by measuring the input voltage to the tuned circuit by means of a selective voltmeter and thereby calculating the effective load on the signal source which had a defined output resistance.

The inductance of the test circuit was a 4 turn, 45 cm diameter loop antenna which was constructed from

a 4 core "mains" cable. Each core consisted of 32 strands and had an overall diameter of 1.5 mm and was covered with 1 mm thick insulation. The antenna had a dc resistance of  $0.1\Omega$  and a self-resonant frequency of 4.6 MHz. The effective antenna loss is therefore within a few per cent of the true loss. The results obtained are shown in Table 1.

The results indicate the superior performance of silver mica, polystyrene and polypropylene capacitors. Polyester, paper and polycarbonate capacitors are not suitable as matching network components for low loss antennae in the frequency range of interest. The increased loss with frequency using the silver mica capacitor is due to the higher antenna loss.

# Coupling loss resistance (R<sub>c</sub>)

The value of this component is the most difficult to calculate since it depends on the physical relationship between the antenna and its surroundings. The magnitude of R<sub>c</sub> can vary between a fraction of an ohm and several ohms and is influenced by frequency and by the material of the coupled object.

For underground mining applications, as was pointed out earlier in this paper, the surrounding rock has little effect on the antenna impedance but when the antenna is indirectly coupled into metallic objects to enhance radio propagation, a greater change in impedance occurs.

In order to determine the magnitude of R<sub>c</sub> due to the presence of conducting objects, the previous experiment was conducted using a silver mica capacitor and the loss measured as the antenna approached a 1 m square copper plate (earth plane).

# Increased antenna loss

The results, which are uncorrected for the antenna self capacitance, are shown in Fig. 6. In free space the antenna self resonant frequency was 4.6 MHz and increased to 6.4 MHz when the antenna was placed 2 mm from the conducting object. Hence it is estimated that a maximum error of 8% is caused by the antenna self capacitance. The results indicate the increase of antenna loss with frequency and that when the antenna is more than 2 cm from a copper plate the loss is effectively equal to the free space loss.

The parasitic capacitances (C<sub>t</sub> and

C<sub>g</sub>) change the antenna impedance | and disturb the direction of current flow. Ideally, all the antenna input current should flow through L in order to produce a well controlled field pattern but the presence of current in C<sub>g</sub> leads to other propagation and resonant modes and a loss in the received signal. The effect of C<sub>t</sub> is not significant if its reactance is greater than that of the inductive reactance. In such a case Ct only modifies the effective antenna inductance and resistance according to equation (2) and hence it can often be absorbed in the matching network.

### Summary of results

Formulae for calculating the value of  $C_{\rm g}$  and  $C_{\rm t}$  have been presented by De Vore and Bohley<sup>2</sup>. By applying these formulae to the four-turn antenna considered in this paper, the values for  $C_{\rm g}$  and  $C_{\rm t}$  were calculated to be 0.2 pF and 40 pF respectively, when the antenna was 1 cm above a ground plane. A better method for assessing the effects of  $C_{\rm g}$  and  $C_{\rm t}$  can be obtained by plotting the antenna impedance using a vector impedance meter over a range of frequencies.

The results indicate that when the antenna is on a wooden table one metre above earth the inductance at 500 kHz is 19  $\mu$ H and the resonant frequency is 4.6 MHz from which the total  $C_t$  (C) is calculated to be 63 pF. For higher frequencies the antenna impedance is at first capacitive and its impedance falls but above 31 MHz the impedance changes periodically every 12 MHz.

In this frequency range the antenna circumference is comparable to the wavelength and the radiation resistance is having a significant effect on the antenna input impedance.

When the antenna is placed on a 1 m-square copper plate, the antenna

inductance at 500 kHz reduces to 6.6  $\mu$ H. The first parallel resonant frequency of the antenna increases to 6.4 MHz from which it is estimated that the total value for  $C_t$  (C) is 81 pF.

Hence both antenna inductance and capacitance are influenced by conducting objects. The results also show that additional resonances occur between 20 MHz and 22 MHz and these are possibly due to the effects of  $C_{\rm g}$ .

It is shown in the previous section that in the frequency range 100 kHz to 3 MHz the antenna is equivalent to a simple parallel tuned circuit whose components are influenced by the presence of conducting objects. This section verifies the equivalent circuit at 913 kHz, which is near the highest frequency extremes used in underground applications, by comparing the practical and theoretical values of the matching network input impedance when fed from a loop antenna.

The matching network used for the experiment is shown in Fig. 7. The output impedance (Z<sub>o</sub>) was measured at a number of frequencies (f) using a vector impedance meter. The theoretical values were calculated.

The matching network components for Fig. 9 were calculated using the antenna's effective inductance and loss since these are easily obtainable.

Using practical silver mica capacitors with  $C_2 = 1.507$  nF;  $C_3 = 16.46$  nF, the variation of the magnitude and phase of  $Z_0$  with frequency is shown in Figs. 8 and 9 respectively.

Two theoretical curves are plot-

Fig. 8: Variation of the magnitude of  $Z_0$  with frequency

Fig. 9: Phase variance of  $Z_0$  with frequency

ted, namely, the solid curve which uses the actual values L and R, these being calculated from equation (2) using  $L_e$  and  $R_e$  with  $C_1$  being derived from the antenna impedance curve, while the dashed curve used the measured values for L and R, that is,  $L_e$  and  $R_e$ .

### **Design difficulty**

Very good agreement with the theoretical results occurs when L, R and  $C_1$  are used. However, if  $L_e$  and  $R_e$  are used and  $C_1$  assumed to be zero, then the theoretical results are shifted to a higher frequency.

These results indicate that the maximum impedance occurs at a slightly higher frequency than zero phase angle which is inductive for frequencies between the series and parallel resonant modes but which is capacitive elsewhere. The variation of the impedance components would be troublesome if the matching network is followed by an RF filter. Such a filter is difficult to design when the source impedance parameters are as complex as those shown in Fig. 8 and Fig. 9.

In Part 2 of this article the equivalent circuit is used to compare theoretically various matching networks and to show which is the desired network for electrically small loop antennas.

# References

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- 2. De Vore, R. and Bohley, P. "The electrically small magnetically loaded multi-turn loop antenna". IEEE Transactions on antennas and propagation, Vol AP-25, pp 496-505, July 1977.

