CHAMBER OF MINES OF SOUTH AFRICA
RESEARCH ORGANISATION

REPORT ON

SOME CONSIDERATIONS REGARDING THE CHOICE OF A
MATCHING NETWORK FOR ELECTRICALLY
SMALL LOOP ANTENNAS

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TECHNICAL SUMMARY

This report presents a comparison of the three capacitive matching networks most commonly used with electrically small loop antennas. The three networks are considered as special cases of a universal capacitive matching network which together with the antenna is analysed by means of a desk-top calculator. Graphical plots of the important parameters indicate the simple series tuned circuit to have many important advantages. Furthermore, narrowband radio frequency modulation techniques are shown to be desirable since the antenna gain and selectivity increase with antenna Q factor and hence causes a reduction of bandwidth.
1. **Introduction**

Electrically small loop antennas are used where medium and high frequency radio transmission is required using an antenna having a minimum volume. These antennas are suited to underground mining situations and are being used for voice, telemetry and remote control applications. However, the small electrical size of the antenna leads to a relatively large reactive input impedance which is equalised with a matching network in order to optimise the efficiency of the transmission system. Generally, the matching network is designed to operate at resonance but frequently the transmitted signal spectrum requires the antenna and matching network characteristics to be known over a range of frequencies centred on the resonant frequency. This is of particular interest for the transmission of digital telemetry data.

This report presents an analysis of an antenna and matching network system in order to assist in the design of such a system. The analysis is extended to the three capacitive matching networks most commonly used with electrically small loop antennas whose properties are compared to establish which is the most suitable. Use is made of a desk-top calculator to perform the calculations and plot the results and the program developed is listed in the Appendix.

2. **A universal capacitive matching network**

Most matching networks use the simple two element "L circuit" which can transform a large range of load impedances to a real input impedance. The eight forms of this circuit consist of various combinations of inductors and capacitors and have been examined by P.H. Smith, who presented a series of graphs to enable the components to be selected. However, G.S. Smith, has shown that the most efficient coupling networks for an inductive antenna consist only of capacitors and a network which incorporates all the various capacitive matching circuits is shown in Figure 1.

2.1 **Circuit analysis**

Three cases of the network in Figure 1 are of importance, these are:
Figure 1
Universal capacitive matching network
(A) \( C_1 = 0 \)

(B) \( C_3 = 0 \)

(C) \( C_1 = C_3 = 0 \)

The matching networks corresponding to the above are denoted by (A), (B) and (C) and it is evident that (A) and (B) are two of the eight forms of "L circuits". Network (C) has not been considered by P.H. Smith but it will be instructive to analyse this series tuned circuit since it has many advantages. Generally, a matching transformer is used with (C) since its input impedance at resonance is low. However, the transformer will be omitted for the present calculations since its only effect is to change the magnitudes of the source impedance and source voltage to the equivalent values shown in Figure 1.

When the antenna is in the receiving mode, \( R_T \) is the load impedance \( R_L \) "seen" by the matching network, \( V_i \) is the induced voltage and \( V_s \) is zero. In the transmit mode \( V_i \) is zero and \( R_T \) is equal to the source impedance \( R_S \).

The most important parameters in a matching network are the magnitude and phase of the antenna current \( I_A \), the loaded output voltage \( V_L \) and the input impedance \( Z_i \). Each of these is derived in the Appendix and can be expressed by the following equations:
\[ \frac{I_A}{V_s} = \frac{z_3 + jz_4}{\left\{z_1 + R_s(z_3 - y_1) + j[z_2 + R_s(z_4 + y_2)]\right\}} \cdot \{z_6 + jz_7\} \]  

\[ \frac{V_L}{V_i} = \left\{\frac{y_3}{y_4 + jy_5}\right\} \cdot \left\{\frac{[z_3 - y_1 + jR_L(z_4 + y_2)]}{[R_L(z_3 - y_1) + z_1] + j[R_L(z_4 + y_2) + z_2]}\right\} \]  

\[ z_i = \frac{z_1 + jz_2}{(z_3 - y_1) + j(z_4 + y_2)} = R_i + jX_i \]  

where \( z_1 = 1 - \omega^2 L(C_1 + C_2) \) \( z_2 = \omega R(C_1 + C_2) \) \( z_3 = -\omega^2 C_1 C_2 R \) \( z_4 = \omega C_2 (1 - \omega^2 L C_1) \) \( z_5 = 1 - \omega^2 L C_1 \) \( z_6 = \omega C_1 R \) \( z_7 = \omega C_1 R \) \( y_1 = \omega C_3 z_2 \) \( y_2 = \omega C_3 z_1 \) \( y_3 = 1 - \omega^2 L C_4 \) \( y_4 = \omega C_4 R \) \( y_5 = \omega C_4 R \)  

\( y_3 = \frac{c_2}{c_2 + c_3} \) \( c_4 = c_1 + \frac{c_2 c_3}{c_2 + c_3} \) \( \omega = 2\pi f \)  

3. The selection of the matching network components

Graphs, which enable the matching components to be selected when the magnitude of the antenna impedance is of the same order as the terminal impedance, have been presented previously by P.H. Smith. However, when the antenna resistance is a small fraction of the required terminal impedance, other methods of selection are necessary.
For matching network (A) the component selection procedure is done in two steps. The first step requires the antenna loss \( R \), antenna inductance \( L \) and antenna capacitance \( C_1 \) to be known together with the required output impedance at resonance \( f_0 \). The network output impedance at \( f_0 \) can be derived from equation (3), which becomes:

\[
\left( \frac{2\pi f_0 L}{R} \right)^2 \left( \frac{C_2}{C_2 + C_3} \right)^2 + \frac{1}{j2\pi f_0 C} \]  

(4)

where \( C = C_3 \left[ 1 + \frac{C_1 C_2}{C_3 (C_1 + C_2)} \right] \).

For many applications the reactive term can be ignored and the output impedance becomes:

\[
\left( \frac{2\pi f_0 L}{R} \right)^2 \left( \frac{C_2}{C_2 + C_3} \right)^2 = R_o \]  

(5)

The second step in the selection of \( C_2 \) and \( C_3 \) makes use of the resonance equation, that is,

\[
\frac{1}{(2\pi f_0)^2} L = C_1 + \frac{C_2 C_3}{C_2 + C_3} = C_r \]  

(6)

Hence the values of \( C_2 \) and \( C_3 \) can be found by solving equations (5) and (6).

The component selection for matching network (B) uses the variation of the effective inductance \( L_e \) and loss \( R_e \) of a parallel tuned circuit. As the frequency approaches resonance both \( L_e \) and \( R_e \) increase. The component values are selected such that \( R_e \) equals the desired output resistance and \( C_2 \) is chosen to resonate the resulting value of \( L_e \). The selection of \( C_1 \) and \( C_2 \) is therefore done in three steps. The first step requires \( R, L, f_0 \) and \( R_o \) to be
known and hence $C_1$ is solved using:

$$R_o = \frac{R}{[1 - (2\pi f_o)^2 LC_1]^2 + (2\pi f_o C_1 R)^2} \tag{7}$$

The value for $C_1$ is easily calculated by using a computer to plot $R_o$ against $C_1$. It should be noted that $C_1$ includes the antenna self-capacitance.

The second step evaluates $L_e$ from:

$$L_e = \frac{L [1 - (2\pi f_o)^2 LC_1] - C_1 R^2}{[1 - (2\pi f_o)^2 LC_1]^2 + (2\pi f_o C_1 R)^2} \tag{8}$$

The third part resonates $L_e$ with $C_2$ and hence:

$$C_2 = \frac{1}{(2\pi f_o)^2 L_e} \tag{9}$$

The selection of the components for matching network (C) requires the effective antenna inductance ($L_e$) and loss ($R_e$) to be known, whereupon, $C_2$ is chosen to resonate with ($L_e$) and the value for ($R_e$) determines the turns ratio for the matching transformer.

4. Basic properties of various capacitive matching networks

In this section the magnitude and phase of the antenna current, loaded output voltage and input impedance are compared for the three matching networks under consideration to establish the most suitable network. The networks are analysed using the computer program which is listed in Appendix 9.4. The program is written in H.P. Basic for a Hewlett-Packard desk-top calculator (9830A). The results are drawn by means of a Hewlett Packard plotter (9862A).
4.1 The network components \((C_1, C_2, C_3)\)

The procedures outlined in the previous section were used to calculate \(C_1\), \(C_2\), and \(C_3\), for each of the matching networks to resonate an antenna at 913 kHz.

The output impedance required from (A) and (B) is 50 \(\Omega\) and from (C) 1.23 \(\Omega\), which is equal to the loss resistance \(R\) of a particular antenna having an inductance of 21.4 \(\mu\text{H}\); the antenna is assumed to have no self capacitance. Hence the antenna Q factor \(\frac{\sqrt{L}}{R}\) is 100.

(A) \(C_1 = 0; C_2 = 1.52 \text{ nF}; C_3 = 22.30 \text{ nF}\)

(B) \(C_1 = 1.20 \text{ nF}; C_2 = 221 \text{ pF}; C_3 = 0\).

(C) \(C_1 = 0; C_2 = 1.419 \text{ nF}; C_3 = 0\).

4.2 The normalised antenna current \((I_A)\)

The variation of the antenna current with normalised frequency \((\frac{f}{f_0} = 913 \text{ kHz})\) for the three networks when driven by matched source impedance is shown in Figure 2. The curves for networks (B) and (C) have identical shapes although (C) is displaced 16 dB above (B). This result is not surprising since no matching transformer is used with (C) which has an input resistance \(R_1\) at resonance of 1.23 \(\Omega\). If all the input power is dissipated in the antenna resistance \(R\), and \(Z_i\) at resonance is purely resistive then:

\[
\frac{V_i^2}{4R_1} = I_A^2 R
\]
Hence for identical antennas and voltage sources the ratio of the resonant currents in (B) and (C) are:

\[
\frac{I_1}{I_1(C)} = 16^\frac{1}{2}
\]

which, expressed logarithmically is equal to 16.1 dB; which is the difference between the two responses in Figure 2.

Network (C) with a perfect matching transformer therefore produces an equivalent maximum antenna current as in (A) and (B). However, the frequency response using network (A) is not quite the same, being displaced to a lower frequency range. A factor common to all curves is that the frequency difference at the 3 dB points \(f_2 - f_1\) is given by half the antenna Q factor, i.e.

\[
f_2 - f_1 = \frac{2f_0}{Q}
\]

The above relationship holds only for a source impedance equal to the matching network input impedance at the desired resonant frequency. The effect of other values of source impedance is treated later in this report.

4.3 The normalised loaded output voltage \(\frac{V_L}{V_i}\)

The shape and position relative to the frequency axis of the normalised output voltage frequency characteristics for networks (B) and (C) are identical to those in Figure 2 if \(R_L = R_S\). The shape of the characteristic for network (C) is also the same as that in Figure 2 but the maximum output is 16 dB less than (B) or (C). This is due to the difference of terminal impedances, since no matching transformer is used for the calculations. Again, the response from (A) is shifted to a lower frequency range than either (B) or (C) and the bandwidth of all three is determined by Q/2.
Electrically small antennas therefore exhibit the property that the system gain and selectivity can be controlled by means of the antenna Q factor.

The effect of Q on the system output can be deduced easily if it is assumed that each network transfers the same input power to the load. Using these considerations then $I_A$ and $V_L$ are each proportional to $Q^k$ and for identical receiving and transmitting antennas the total system voltage gain is proportional to $Q$.

The frequency response of (A) and (B) over a wide frequency range is shown in Figure 3. The response from (C) is not shown since it is identical to (B). It is apparent that (A) exhibits more outband attenuation than (B) above the resonant frequency but below the resonant frequency (B) is superior. For receiver applications these points are important since the noise performance of the system is influenced by the matching network selectivity.

The ideal frequency responses obtained for $I_A$ and $V_L$ imply that $I_A$ and $V_L$ are the reciprocals of each other when the terminal impedances are the same. Complete reciprocity also requires the phase responses to be identical and this has been found to be true. The phase response for (A) and (B) is shown in Figure 4. The phase response of (B) is identical to that of (C). The phase response for all three networks is linear over the system bandwidth which indicates that these antennas should give good performance in digital transmission systems.
Figure 3

The loaded output voltage over a wide frequency range
The variation of loaded output voltage phase with frequency
The input impedance \( Z_i \)

The variation of \( Z_i \) with frequency for (A) and (B) and for (C) with a perfect matching transformer is shown in Figure 5; the phase responses are shown in Figure 6. It is apparent that (B) and (C) are similar since the impedance increases for frequencies away from resonance while the impedance for (A) decreases. An extended plot reveals that although (B) and (C) are similar within the system bandwidth, the difference between the two increases as the frequency deviates further from resonance. The phase responses for (B) and (C) are also different, both being capacitive below resonance, with (C) becoming inductive above resonance while (B) is at first inductive and then capacitive. Network (A) behaves in a similar manner to (B) but the inductive region is below resonance.

It is important to bear in mind that the impedance curves in Figure 5 and Figure 6 refer to one set of antenna specifications and that different results can be obtained for other antenna systems. In particular it is not always possible to obtain a real value of input impedance and Smith\(^1\), has shown that it is not possible to match all types of antenna impedance with networks (A) and (B). For example, if network (A) is designed for an output impedance of 5 \( \Omega \) with the antenna in Section 4.1, the phase response is shifted such that the peak just reaches zero for \( Q = 100 \) at a normalised frequency of 0.995. For all other frequencies the input impedance is capacitive. If the output impedance is 50 \( \Omega \) the critical Q factor is 10. In these cases the capacitive term in equation (4)
Figure 5

The variation of matching network input impedance with frequency
Figure 6
The variation of the phase of the matching network input impedance with frequency
has a significant effect. If the Q factor for this antenna is reduced from 100 for an output impedance of 5 Ω the impedance is always capacitive and additional components are required to produce a real input impedance and hence obtain efficient power transfer.

A restriction on the range of antenna impedance that can be matched is also imposed by network (B). However, network (C) can always provide a real input impedance and furthermore, its impedance increases away from the resonant frequency. This is a desirable property for transmitter applications since a large part of the signal spectrum can be at the extremes of the system bandwidth and the supply current will be less for an impedance characteristic which increases rather than decreases. Network (A) however, exhibits a 60% decrease in impedance at the lower edge of the band with a consequent increase in current consumption.

The asymmetric decrease of impedance about resonance for network (A) is due to the series resonance of L and C2 which can cause the input impedance to decrease to a very low value within the system bandwidth. The rate of decrease increases as the ratio \( \frac{C_2}{C_3} \) decreases, that is, for high inductance antennas or networks with a low output impedance. With some antenna systems a decrease of 80% at the lower 3 dB point has been calculated. The change of \( Z_i \) with frequency can cause a problem with portable equipment such as speech transceivers if variations in the matching or antenna components cause the resonant frequency to increase by greater than half the system bandwidth. In this situation the operating frequency could coincide with the impedance minimum and the
operation of the radio transmitter and the RF filter of the radio receiver may be adversely affected.

A close examination of Figure 2 and Figure 5 reveals that the impedance peak for network (A) is at a higher frequency than the peak of the loop current. As the antenna Q factor reduces, the frequency separation between the two peaks increases and with the antenna used for this example, if the Q factor is less than 20, the loop current at the frequency of the impedance maximum is less than the resonant peak by more than 2 dB. For an output impedance of 5 Ω the critical Q factor is 50. Consequently the output impedance peak is not an indication of resonance and if used for tuning the antenna it can cause misleading results. Conversely the output impedance may be significantly less at resonance than the maximum value and hence overload the radio transmitter.

These factors are important for intrinsically safe equipment since in these applications the antenna voltage is limited and a low impedance is often used to obtain higher transmission powers.

5. The effect of component tolerances on the frequency response

The previous sections show that a high antenna Q factor leads to a higher input signal at the radio receiver and to better selectivity. However, before a value for the Q factor can be determined, it is necessary to consider the sensitivity of the system parameters to practical variations in the components of the antenna and matching network.
The maximum change of frequency response allowable depends on the type of transmission system. If the modulation bandwidth is much less than the transmission bandwidth then the maximum change of resonant frequency is generally taken as half the transmission bandwidth. If the two bandwidths are comparable then the resonant frequency drift must be much less and depends on the type of modulation. For example, the minimum transmission bandwidth for a digital system is comparable to the baud rate\(^3\). In this case the frequency stability should be better than one tenth\(^4\) of the data rate and for a resonant frequency tolerance of \(\pm 0.5\%\) the antenna Q factor must be less than 40. For the purposes of this paper the component tolerances are considered for a system whose modulation bandwidth is much less than the transmission bandwidth.

5.1 Capacitance effects

Generally the antenna capacitance is much smaller than the capacitance of the matching components and hence its variation due to temperature is a second order effect. The antenna capacitance is also influenced by the presence of conducting objects which affect the antenna's inductance as well. However, below the antenna self-resonant frequency the antenna capacitance can be absorbed in the inductance and the effect of the two are treated in the next section.

The values of the matching network components, however, change with temperature, the environment and age by amounts which depend on the type of components used. It is recommended that silver mica capacitors be used for receiver applications and polypropylene capacitors for transmitter matching networks. Silver mica capacitors have a long-term stability of about 0.1% and a temperature coefficient of \(\pm 50 \text{ ppm per } ^\circ\text{C.}\)
Hence a typical design tolerance for these components is ± 0.2%. Polypropylene is not as stable (0.5%) as silver mica and has a higher temperature coefficient (-200 ppm per °C), and hence a typical design tolerance for these components is about ± 1%.

The effect of capacitance variations on matching network (A) depends on the relative magnitudes of the components. For most applications the antenna impedance at resonance is much greater than the network output impedance and hence $C_3$ is significantly greater than $C_2$.

Therefore, the resonance condition will be influenced primarily by $C_2$ rather than $C_3$.

For reasons mentioned previously this network is more suitable for receiver applications in which case silver mica capacitors would be used. Assuming a tolerance of ± 0.2%, the resonant frequency changes by ± 0.1% (since $\omega_0 = (LC)^{-1/2}$) which implies that for this change in component value to be serious the antenna Q factor would be in the region of 1 000, an impractical situation. Furthermore, the output impedance of the network is proportional to $\left(\frac{C_2}{C_3}\right)^2$ and hence a worst case error of ± 0.8% is expected and this is also generally insignificant.

With network (B), however, the component sensitivity is not quite so clear since the parallel capacitance ($C_1$) has a complex effect on $L_e$ and $R_e$. If $R_e$ is plotted against $C_1$ at 913 KHz then Figure 7 shows that large values of $R_e$
Figure 7

The effects of $C_1$ on the effective resistance of a parallel tuned circuit
are very sensitive to $C_1$. Even when $R_e = 50 \ \Omega$, the effective resistance changes by about 10% for a 1% change of $C_1$. The change of $L_e$ is not quite so dramatic, being about half the change of $R_e$ and hence for a 1% change of $C_1$, the series resonant frequency will change by about 2.5%. Hence the $Q$ factor should be less than 40 in order to achieve a stable system.

The effect of $C_2$ using network (C) is straightforward. In practice this network is suited to transmitter applications and hence the maximum change of resonant frequency with polypropylene capacitors should not exceed 0.5% and the $Q$ factor must be less than 200.

5.2 Inductance effects

The antenna inductance is hardly affected by changes in temperature. However, its value is affected by other factors such as the presence of conducting objects and by physical movement if the antenna is flexible and is moved. Measurements at 200 kHz indicate that the inductance can be reduced by 7% when indirectly coupled within a few centimetres to underground power-line cables. The inductance of a portable antenna worn across the chest may change by $\pm$ 2% due to movement of the body.

If the equipment is portable then the maximum change of frequency response must lie within the transmission bandwidth. With network (A) the antenna $Q$ factor should be less than 100 for the majority of mining applications and the transmitter must be able to deliver power into the reduced matching network input impedance as the impedance response drifts. The antenna $Q$ factor using network (C) should also be less than 100 to avoid retuning but, although the transmitter will generally not be harmed by a
load impedance higher than nominal, it would limit the maximum output voltage with a consequent increase in transmitted harmonics. The effect of an inductance change using network (B) is far more serious and Figure 7 indicates that for a 30% reduction of \( L \), \( R_e \) falls from 50 \( \Omega \) to 6 \( \Omega \). For a 2% reduction of \( L \) both \( R_e \) and \( L_e \) reduce by almost a half and complete retuning is necessary.

In some permanent installations, such as the transmission of telemetry data in shafts, indirect coupling into power-line cables is used to achieve a considerable increase in received signal strength. The received signal can be increased further by tuning the antenna in-situ and practical measurements indicate that 100% improvement can be obtained. Using network (A), \( C_2 \) is adjusted to bring the resonant peak for either \( I_A \) or \( V_L \) coincident with the frequency of operation. Using this procedure \( Z_0 \) does not change since its real part reduces to:

\[
\frac{1}{w^2 C_3 R}
\]

However, with \( L \) smaller than the free space value, the antenna \( Q \) factor is reduced and the transmission bandwidth is increased. Retuning using network (C) is also simple but with network (B) both matching components must be adjusted by a fairly complex procedure.

5.3 Source and load impedance effects

The effect of source impedance (\( R_s \)) on the antenna current (\( I_A \)) using network (A) is shown in Figure 8.
Figure 8
The effect of source resistance on the loop current frequency response using network (A).
As the source impedance decreases from 500 Ω the bandwidth widens with the response at first being centred just below 913 kHz and then moving to lower frequencies. For values of source impedance less than 2 Ω, the current maximum occurs at the series resonant frequency of the network. The bandwidth increases until the source impedance is about 5 Ω, whereafter it decreases. When $R_s = 5$ Ω the bandwidth is double the value for a matched system. The real and imaginary parts of the antenna input impedance at the maximum value are given in Table 1.

**TABLE 1**

<table>
<thead>
<tr>
<th>$R_s$ (Ω)</th>
<th>$R_{iA}$ (Ω)</th>
<th>$X_{iA}$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>1.41</td>
<td>0.51</td>
</tr>
<tr>
<td>5.0</td>
<td>2.44</td>
<td>2.91</td>
</tr>
<tr>
<td>20</td>
<td>31.18</td>
<td>18</td>
</tr>
<tr>
<td>50</td>
<td>51</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>51</td>
<td>0</td>
</tr>
</tbody>
</table>

From this information it is possible to calculate, for a lossless matching network, how much extra reactive power must be supplied for the same power into $R_{iA}$ and hence the same antenna current. The calculated values are given in Table 2.

**TABLE 2**

<table>
<thead>
<tr>
<th>$R_s$ (Ω)</th>
<th>Extra transmitter power (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.27</td>
</tr>
<tr>
<td>5</td>
<td>1.92</td>
</tr>
<tr>
<td>20</td>
<td>0.62</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>0</td>
</tr>
</tbody>
</table>
With some transmitter circuits the output impedance can be adjusted independently of the output power. If the output impedance of such a transmitter is increased above 50 Ω while keeping the output power constant, the system bandwidth reduces but the maximum antenna current remains the same. This is not a desirable mode of operation since the reduced bandwidth could be obtained with a higher antenna Q factor with a consequent increase in the maximum antenna current. It is therefore concluded that the optimum source impedance for network (A) is equal to the network input impedance.

The effect of source impedance variations on networks (B) and (C) is to change only the system Q factor; the resonant peak remains at the same frequency. As the source impedance is reduced, the responses become narrower and hence the effective antenna Q factor increases. For an antenna of a defined number of turns this property can be used to advantage to increase the antenna current while keeping the system Q factor constant. In order to achieve this it is necessary to have the greater part of the total series resistance which determines the system Q factor in the output impedance of the transmitter where, by suitable circuitry, it absorbs little power. As the antenna loss (R) decreases, the transmitter output impedance must increase to keep the system Q factor constant, but for a transmitter of defined output power ($P_o$) the antenna current is given by:

$$I = \frac{P_o}{R}$$

Hence by using a low loss antenna and a matching transformer with a high turns ratio, it is possible to obtain
a high value of antenna current. An antenna using these principles has been built and has produced a 200 μV (p.d.) signal at a receiver antenna when indirectly coupled to 2.5 km of 6.6 kV power cable. The transmitter had an output power of 10 W.

6. Matching network comparison

Network (A) is used frequently for matching loop antennas since it is simple to design and being only capacitive, is highly efficient. However, for transmitters it has the disadvantage of a sharp reduction of input impedance on the low-frequency side of the resonance curve and thereby increases the consumption of current by the transmitter, if the modulation bandwidth is comparable to the transmission bandwidth. In radio receiving systems it has the disadvantage of lower outband attenuation compared to the other matching networks for frequencies below resonance which is a region that generally has a greater noise content. In situations where the antenna Q factor is low it is difficult to provide a terminal impedance which is real and which has a low value. The optimum source impedance is equal to the input impedance of the network and its value should be tightly controlled if a real input impedance is required at the system resonant frequency.

Matching network (B) produces an identical frequency response to network (C). However, network (B) is restricted in cases of antenna detuning since both \( L_e \) and \( R_e \) are sensitive to capacitance and inductance variations. Typical long-term stability and temperature variations of the matching components restrict the antenna Q factor to less than 40.
If retuning is necessary then both matching components must be adjusted which is a difficult task in the field. However, this network has an increasing impedance characteristic away from resonance which is a desirable characteristic for transmitter applications.

The problems existing with matching networks (A) and (B) are not present with (C). Since only one resonating component is used, the task of retuning is much simpler than with (A) or (B). The impedance characteristics increases at frequencies away from resonance and its lower outband attenuation is better than that of (A). Furthermore, it is theoretically possible to match the antenna to any real value of terminal impedance. However, it has the disadvantage of requiring a transformer to match the low-impedance antenna to a transmission line and for a high turns-ratio the design of a low-loss transformer is no easy task.

7. Conclusion

This report has presented an analysis of an electrically small loop antenna together with a universal capacitive matching network. Various matching networks have been compared and it has been shown that the simple series resonant circuit possesses many important features.

The resulting frequency responses indicate that the best performance is achieved with a narrow band transmission system using an antenna with a high Q factor. The maximum Q factor is determined partly by the effect of the mine environment on the maximum change of all the components of the antenna system, by the distortion which may be introduced into the demodulated radio frequency signal and by the decreasing noise performance of some modulation methods which have a narrow bandwidth.
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APPENDIX

1.1 The universal matching network input impedance \((Z_1)\)

In Figure 1 the impedance between 2 and 3 without \(C_3\) and the termination is given by:

\[
Z_{23} = \frac{(R + j\omega L)\frac{1}{j\omega C_1}}{R + j\omega L + \frac{1}{j\omega C_1}} + \frac{1}{j\omega C_2}
\]

\[
= \frac{R + j\omega L}{1 - \omega^2 LC_1 + j\omega C_1 R} + \frac{1}{j\omega C_2}
\]

\[
= \frac{1 - \omega^2 L(C_1 + C_2) + j\omega R(C_1 + C_2)}{-\omega^2 C_1 C_2 R + j\omega C_2(1 - \omega^2 LC_1)}
\]

\[
= \frac{Z_1 + jZ_2}{Z_3 + jZ_4}
\]

where \(Z_1 = 1 - \omega^2 L(C_1 + C_2)\); \(Z_2 = \omega R(C_1 + C_2)\)

\(Z_3 = -\omega^2 C_1 C_2 R\); \(Z_4 = \omega C_2 (1 - \omega^2 LC_1)\)

The impedance between 2 and 3 including \(C_3\) but without the termination is given by:

\[
Z_1 = \frac{Z_{23} \frac{1}{j\omega C_3}}{Z_{23} + \frac{1}{j\omega C_3}}
\]

(2)

Substituting for \(Z_{23}\) from (1) into (2) gives:

\[
Z_1 = \frac{Z_1 + jZ_2}{(Z_3 - \omega C_3 Z_2) + j(Z_4 + \omega C_3 Z_1)}
\]

(3)

or

\[
Z_1 = \frac{Z_1 + jZ_2}{(Z_3 - Y_1) + j(Z_4 + Y_2)}
\]

(4)

where \(Y_1 = \omega C_3 Z_2\); \(Y_2 = \omega C_3 Z_1\)
1.2 The loaded output voltage ($V_L$)

The loaded output voltage can be derived by using Thevenin's equivalent circuit. First the open circuit output voltage ($V_{oc}$) and output impedance ($Z_l$) are found and then used to obtain the loaded output by means of the potential divider formed by $Z_l$ and $R_L$. Using Figure 1 with no load, then:

\[
\frac{V_{13}}{V_i} = \frac{1}{\frac{1}{j\omega C_4} + \frac{1}{j\omega C_4}} = \frac{1}{1 - \frac{1}{\omega^2 LC_4 + j\omega C_4 R}}
\]

where, $C_4 = C_1 + \frac{C_2 C_3}{C_2 + C_3}$

therefore, \[
\frac{V_{oc}}{V_i} = \frac{C_2}{C_2 + C_3} \quad \frac{V_{13}}{V_i} = \frac{C_2}{C_2 + C_3} \left(\frac{1}{1 - \frac{1}{\omega^2 LC_4 + j\omega C_4 R}}\right)
\]

\[
= \frac{Y_3}{Y_4 + jY_5}
\]

where, $Y_3 = \frac{C_2}{C_2 + C_3}$; $Y_4 = 1 - \frac{1}{\omega^2 LC_4}$; $Y_5 = \omega C_4 R$

From equation 4 in Appendix 1.1.

\[
Z_l = \frac{Z_1 + jZ_2}{Z_3 - Y_1 + j(Z_4 + Y_2)}
\]

Hence if: \[
\frac{V_L}{V_{oc}} = \frac{R_L}{R_L + Z_l}
\]

then: \[
\frac{V_L}{V_i} = \left[\frac{Y_3}{Y_4 + jY_5}\right] \left[\frac{R_L}{(Z_3 - Y_1) + jR_L(Z_4 + Y_2)}\right] \left[\frac{R_L}{Z_1 + jR_L(Z_4 + Y_2) + Z_2}\right]
\]
1.3 The antenna current ($I_A$)

The input voltage to the matching network of Figure 1 in the "transmit" mode is given by:

$$V_{23} = V_s \frac{Z_i}{Z_i + R_s}$$

(5)

The current flowing through $C_2$ ($I_{12}$) can be found using the value for $Z_{23}$ derived in Appendix 1.1. that is:

$$I_{12} = \frac{V_{23}}{Z_{23}}$$

$$= V_s \left( \frac{Z_i}{Z_i + R_s} \right) \left( \frac{Z_3 + jZ_4}{Z_1 + jZ_2} \right)$$

(6)

Substituting the value for $Z_i$ derived in Appendix 1.1., equation (6) becomes:

$$I_{12} = \frac{(Z_3 + jZ_4)}{Z_1 + jZ_2 + R_s [ (Z_3 - Y_1) + j(Z_4 + Y_2) ]}$$

(7)

Using $I_{12}$, then $I_A$ is given by:

$$I_A = \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + R + j\omega L} = \frac{I_{12}}{1 - \omega^2 L C_1 + j\omega C_1 R}$$

$$= \frac{I_{12}}{Z_6 + jZ_7}$$

(8)

where $Z_6 = 1 - \omega^2 L C_1$; $Z_7 = \omega C_1 R$
Hence by substituting equation 7 into equation 8

the antenna current is given by:

\[
\frac{I_A}{V_s} = \left( \frac{Z_3 + jZ_4}{Z_6 + jZ_7} \right) \frac{1}{R_s(Z_3 - Y_1) + Z_1 + j[R_s(Z_4 + Y_2) + Z_2]}
\]

1.4 Antenna parameter programme
AERIAL PARAMETER PROGRAMME

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DIM A$[100], B$[100], C$[26]

DISP "XMIN, XMAX, SCALE INC."

INPUT A, B, C

DISP "PLOTTING STEP"

INPUT S

DISP "YMIN, YMAX, SCALE INC."

INPUT A1, B1, C4

DISP "X-OFFSET, Y-OFFSET"

INPUT D1, D2

DISP "ASPECT RATIO \(H/W\)"

INPUT A9

DISP "X-AXIS LABEL"

INPUT A$

DISP "Y-AXIS LABEL"

INPUT B$

REM ***********INPUT OF PLOTTING PARAMETERS SECTION***********

REM ***************PLOTTING AND LABELING OF AXIS SECTION**********
- 33 -

710 PLOT -(LEN(B$)/4)+3
715 PLOT -(LEN(B$)/4),0
720 LABEL (+2.5171,P1/2,A9)$[1,(LEN(B$))];
725 PLOT B$+1,1
730 DISP
735 REM
740 REM**********************************************************************
745 REM INPUT OF VARIABLES SECTION******************************************
750 DISP "INSERT VARIABLES C1,C2,C3"
755 INPUT C1,C2,C3
760 DISP "Q FACTOR"
765 INPUT Q
770 DISP "FREQUENCY (HZ)"
775 INPUT W
780 DISP "INDUCTANCE"
785 INPUT L
790 DISP "LOAD RESISTANCE"
795 INPUT R
800 REM
805 REM**********************************************************************
810 REM**********************************************************************
815 PRINT "FUNCTION NAME DESCRIPTION"
820 PRINT
830 PRINT "IMPMAQ ......... IMPEDANCE MAGNITUDE"
835 PRINT "IMPHASE ......... IMPEDANCE PHASE"
840 PRINT "LOADOUTMAQ ......... LOADED OUTPUT MAGNITUDE"
845 PRINT "OUTPHASE ......... LOADED OUTPUT PHASE"
850 PRINT "LOOPMAQ ......... TRANSMIT LOOP CURRENT MAGNITUDE"
855 PRINT "LOOPHASE ......... TRANSMIT LOOP CURRENT PHASE"
860 PRINT
865 DISP "WHICH FUNCTION REQUIRED"
870 INPUT C$
875 IF C$="IMPMAQ" THEN 1200
880 IF C$="IMPHASE" THEN 1380
885 IF C$="LOADOUTMAQ" THEN 1640
890 IF C$="OUTPHASE" THEN 1890
895 IF C$="LOOPMAQ" THEN 2190
900 IF C$="LOOPHASE" THEN 2430
905 PRINT
910 GOTO 860
915 STOP
920 REM
925 REM**********************************************************************
930 REM**********************************************************************
935 REM DEFINITION OF ARCTAN ABSOLUTE FUNCTION*****************************
940 DFC "ARCTAN"(Z)
945 S1=SGN(V1)
950 S2=SGN(V2)
955 DEG
960 IF S1=1 AND S2=1 THEN 1090
965 IF S1=1 AND S2=-1 THEN 1100
970 IF S1=-1 AND S2=-1 THEN 1130
975 IF S1=-1 AND S2=1 THEN 1150
980 Z=ATN(ABS(V1/V2))
985 RETURN Z
990 Z=180-ATN(ABS(V1/V2))
995 RETURN Z
1000 Z=180+ATN(ABS(V1/V2))
1005 RETURN Z
1010 Z=360-ATN(ABS(V1/V2))
1015 RETURN Z
1020 STOP
1025 REM
1030 REM**********************************************************************
1035 REM MAGNITUDE OF THE IMPEDANCE SECTION*******************************
1040 F=G
FOR X=A TO B STEP S

100 P=2

150 NEXT X

160 PEN

170 GOTO 2720

190 DFC "IMPHASE"(X)

220 Z1=-((2*PI*X+W)+2)*L*(C1+C2)

230 Z2=((2*PI+1)+2)*W*X*L*(C1+C2)/Q

240 Z3=-((3+2*PI+1)+3)*W*X*L*(C1+C2)/Q

250 Z4=2*PI*X+X*C2*(1-((2*PI+1)+2)*X+Y2)*(W+2)+L*C1))

260 V1=2*PI+X*C3+Z2

270 V2=2*PI+X*C3+Z1

280 Y3=((Z1+2)+(Z2+2))*(Z3-Y1)+2+(Z4+Y2)+2)*0.5

290 RETURN Y3

300 STOP

350 REM-----------------------------IMPEDANCE PHASE SECTION---------------------------

390 P=3

420 FOR X=A TO B STEP S

440 PLOT X,FC"IMPHASE"(X),P.

450 P=2

470 NEXT X

480 PEN

500 GOTO 2720

520 DFC "IMPHASE"(X)

570 Z1=-((2*PI*X+W)+2)*L*(C1+C2)

580 Z2=((2*PI+1)+2)*W*X*L*(C1+C2)/Q

590 Z3=-((3+2*PI+1)+3)*W*X*L*(C1+C2)/Q

600 Z4=2*PI*X+X*C2*(1-((2*PI+1)+2)*X+Y2)*(W+2)+L*C1))

610 V1=2*PI+X*C3+Z2

620 V2=2*PI+X*C3+Z1

630 Y3=((Z1+2)+(Z2+2))*(Z3-Y1)+2+(Z4+Y2)+2)*0.5

650 RETURN Y3

660 STOP

710 REM-----------------------------MAGNITUDE OF LOADED OUTPUT VOLTAGE SECTION----------

740 P=3

770 FOR X=A TO B STEP S

800 PLOT X,FC"LOADOJTMAG"(X),P.

830 P=2

860 NEXT X

870 PEN

910 GOTO 2720

950 DFC "LOADOJTMAG"(X)

980 Z1=-((2*PI*X+W)+2)*L*(C1+C2)

990 Z2=((2*PI+1)+2)*W*X*L*(C1+C2)/Q

1000 Z3=-((3+2*PI+1)+3)*W*X*L*(C1+C2)/Q

1010 Z4=2*PI*X+X*C2*(1-((2*PI+1)+2)*X+Y2)*(W+2)+L*C1))

1020 V1=2*PI+X*C3+Z2

1030 V2=2*PI+X*C3+Z1

1040 Y3=((Z1+2)+(Z2+2))*(Z3-Y1)+2+(Z4+Y2)+2)*0.5

1050 RETURN Y3

1060 STOP
V5=((-2*PI+W+12)*X+C4=L)\times Q
V6=((R+(23-Y1)+Z1)*12+((R+(24+Y2)+Z2)*12)
V7=((Y9*R+12)*((23-Y1)*12+((24+Y2)*12)
V8=U=-LGT((Y7/((Y6+(Y4+12)+(Y5+12)))))
RETURN V6
STOP
REM---------------------
REM********************LOADED OUTPUT VOLTAGE PHASE**********
P=3
FOR X=A TO E STEP S
PLOT X,FC"OUTPHASE"(X),P
P=2
NEXT X
PEN
GOTO 2720
IFC "OUTPHASE"(X)
Z1=1-((2*PI*W+12)*L*(C1+C2))
Z2=((2*PI*12)*L+2)*X*L*(C1+C2)/Q
Z3=((1)*(2*PI*R3)*((O+2)*(C1+C2)*L)/Q
Z4=2*PI*X+WC2*1-((2*PI*12)*X+2)*((W2)*L+C1))
Y1=2*PI+X+WC2*Z2
Y2=2*PI+X+WC3+Z1
C4+C1+(C2+C3)+(C2+C3))
Y4=1-((X+2*PI*W+12)*L+C4)
Y5=((2*PI+W+12)*X+2+C4)*L/Q
Y6+12+Y2
Y2=(23-Y1)
Z5=P1=FC"ARCTAN"(Z)
Z6=V1=Y5
0=V2=Y4
P2=FC"ARCTAN"(Z)
V1=X+(Z4+Y2)+22
V2=R+(23-Y1)+Z1
P3=FC"ARCTAN"(Z)
P4=P1-P2-P3
RETURN P4
STOP
REM---------------------
REM************************TRANSMIT LOOP CURRENT SECTION************
P=3
FOR X=A TO E STEP S
PLOT X,FC"LOOMPAG"(X),P
P=2
NEXT X
PEN
GOTO 2730
IFC "LOOMPAG"(X)
Z1=1-((2*PI*W+12)*L*(C1+C2))
Z2=((2*PI*12)*L+2)*X*L*(C1+C2)/Q
Z3=((1)*(2*PI*R3)*((O+2)*(C1+C2)*L)/Q
Z4=2*PI*X+WC2*1-((2*PI*12)*X+2)*((W2)*L+C1))
Z6=1-((2*PI+W+12)*L*C1)
Z7=((2*PI*12)*X+2)*X*(C1+L)/Q
Y1=2*PI+W*X+C3+Z2
Y2=2*PI+W*X+C3+Z1
L1=(Z3+2)+(Z4+2)
L2=1+((1+P+(23-Y1))*12+((Z2+(P+(24+Y2))*12)
L3=(Z5+2)+(Z7+2)
L4=10*LGT((L1+1000)/((L2+L3))
RETURN L4