Manuel Rocha Medal Recipient

A Relative Moment Tensor Inversion Technique
Applied to Seismicity Induced by Mining

By

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Summary

Studies of source mechanisms of mining-induced seismic events play an important role in understanding the various modes of failure observed around underground excavations and enable the geometry of likely planes of failure to be determined. These planes can be mapped using conventional techniques, for example, geological fracture mapping. However, such an approach is often problematical due to limited access to the site and/or poor exposures (if any) of the failure plane. An added difficulty is that planes of failure often do not follow faults of geological origin, but are related to the geometry of the advancing stope face. For example, the development of face-parallel shear zones ahead of deep-level stope faces. In such cases, the stresses induced by mining dominate over the geological structure in the critical region close to the stope face. Seismic methods therefore have the potential of being a practical method of studying the development of seismic shear zones underground.

Slip on such a failure plane generates a three dimensional elastic wave that propagates through the rockmass, carrying a wealth of information regarding the source rupture process. The ground motions caused by the passage of the wave can be recorded by arrays of sensitive instruments.
called seismometers. These sets of recordings (seismograms) provide the basic data that seismologists use to study these elastic waves as they propagate through the Earth. Conventional seismic analyses provide scalar measurements of the rupture size and intensity. However, through a process known as moment tensor inversion (MTI), the seismograms recorded from a seismic event can be used to calculate a moment tensor that describes the three-dimensional nature of the source mechanism. Interpretation of the moment tensor gives insight into whether the rockmass failed in tension, compression or shear and indicates the direction of movement and the failure plane.

Moment tensor solutions computed using conventional MTI methods are sensitive to noise and may be biased due to systematic errors in the measurements. The primary objective of this study was to develop a robust MTI method to estimate the moment tensors of clusters of seismic events recorded in the underground environment. To achieve this, three ‘hybrid’ MTI methods were developed by the author. These methods involve different iterative weighting schemes designed to enhance the accuracy of the computed moment tensors by decreasing the effect of outliers (data points whose residuals lie ‘far’ from the mean or median error). The additional information required for hybrid methods is obtained by considering a spatial cluster of seismic events and assuming that the waves generated by each event in the cluster follow a similar path through the rockmass and allowing a common ray-path to be assumed. Hence the unknown effect of the heterogeneous rockmass on the waveform is similar for all the events in the cluster.

The final objective was to determine whether the techniques developed could be successfully applied to real data. The hybrid MTI methods using the median and the weighted mean correction were applied to a cluster of 10 events, having remarkably similar waveforms, recorded at Oryx Gold Mine. For comparative purposes, the more conventional absolute method was also applied. The solutions computed using the hybrid MTI with a median correction displayed a distinct improvement after the iterative residual correction procedure was applied, in contrast to the solutions obtained from the absolute method. The radiation patterns and fault-plane solutions from the hybrid method showed a high degree of similarity, and were probably more accurate reflections of reality. These observations are very encouraging and point towards the potential for using the hybrid MTI method with a median correction as a standard processing tool for mine seismicity.

The implications of this work are that a robust method for calculating the focal mechanisms of clusters of seismic events induced by mining activities has been developed. Regular application will lead to a better understanding of rock fracture processes and to improved safety underground.

Keywords: Moment tensor inversion, mining-induced seismicity, source mechanisms, common ray-path, weighting scheme.

1. Introduction

A seismic source radiates elastic waves having a spectrum of wavelengths. For the long wavelength waves, the rupture area and source volume that release strain energy are relatively small and can be approximated as a point source, whereas the shorter wavelengths require finite source models. The seismic moment tensor describes the equivalent forces of a general seismic point source and can be used to describe the seismic radiation pattern and the strength of the seismic point source. Analysis of seismic moment tensors is used to distinguish and quantify (estimate) the relative contributions of various source mechanism processes. For further information on this topic, refer to Aki and Richards (1980) and Gibowicz and Kijko (1994).

Moment tensor inversions have been routinely performed for several years on large earthquakes recorded at teleseismic distances. However, the application of a moment tensor inversion technique to microseismic events induced by the development of underground excavations is a relatively recent innovation. Only a few works related to the use of moment tensor inversion in source mechanism studies of mining-induced
seismic events have been published (e.g. Spottiswoode, 1984; Brawn, 1989), the reason being that the inversion methods are notoriously tedious and prone to errors.

1.1 Factors Influencing the Accuracy of Moment Tensors

Moment tensor inversion (‘MTI’) is one of the best approaches to study the mode of failure of a seismic source, provided that two major assumptions hold. Firstly, it is assumed that the point source approximation is valid (i.e. that the fault plane dimensions are shorter than the wavelength of the seismic waves used in the inversion), and secondly, that the effect of the Earth’s structure on the seismic waves is modelled correctly. If either of these assumptions does not hold, the resultant moment tensor may contain a large non double-couple component, even if the source mechanism is a double-couple (Streliz, 1978; Barker and Langston, 1982). In this study, the spurious non double-couple component is referred to as a ‘false’ component.

At this stage, a brief excursion from the discussion is useful to describe the ‘double-couple’ model. A number of theoretical source models have been proposed to explain the P-wave polarity observations. These models consist of various types of forces acting at a point source, rather than on a fault plane, because the former is much simpler to analyse than the latter (Fig. 1). The double-couple model (Fig. 1e), introduced in the 1960s, is the currently accepted model. The double-couple model consists of two pairs of forces. Each pair consists of forces with equal magnitude and opposite directions, so that the total force is equal to zero. In addition, the torque of each couple about the origin of the axes is of equal magnitude and opposite to one another so that

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**Fig. 1.** Various force types at a point source: (a) a single force; (b) a pair of equal and opposite forces in tension; (c) a pair of equal and opposite forces as a torque about the z-axis; (d) two pairs of forces, where tension and compression are of equal magnitude and are perpendicular to one another; (e) two pairs of forces with torques about the z-axis of equal magnitude and opposite in direction (redrawn from Kasahara, 1981)
the total torque is zero. Any general fault mechanism can be decomposed into a combination of these double-couple forces.

Although the single-couple and double-couple models shown in Fig. 1(d) and (e) are equivalent with respect to their mechanical effect, the latter is the currently accepted model. The acceptance of this model is based on observational and theoretical evidence and a thorough understanding of the radiation pattern of P- and S-waves generated by the two types of sources. In addition, the force system shown in Fig. 1(e) is directly compatible with the tensor formulation for stress and strain used in rock mechanics (e.g. Jager and Cook, 1976; Aki and Richards, 1980).

A further factor contributing to the presence of ‘false’ components in the moment tensor is the quality of the available data. The quality of the MTI and reliability of the results depends to a large extent on the number and quality of data points and on the azimuthal coverage of stations around the source. The GIGO principle (Garbage In, Garbage Out) is of particular relevance because data having poor signal-to-noise ratios have an adverse effect on the moment tensor solutions. This last factor forms the main thrust of this study – the development of robust MTI methods to accurately estimate the moment tensor in the underground environment.

The effects of various types of noise on moment tensor solutions are briefly demonstrated, using both synthetic and recorded data, to highlight the need for noise reduction techniques. Noisy data have a considerable effect on the MTI and results in the appearance of ‘false’ components in the solutions. Other unwanted effects are blurring of the radiation pattern and variations in the fault-plane solutions. Figure 2(a) shows the radiation pattern and fault-plane solutions resulting from the absolute MTI of synthetic data of a pure double-couple source having a strike of 20°, dip of 60° and rake of 0°. (The various inversion types are discussed in Section 1.2). The fault-plane solutions and three quantities, %ISO, %DC and %CLVD, describing the source mechanism are calculated from the moment tensor and are listed in the figures. %ISO is the percentage isotropic component, %DC is the percentage double-couple component and %CLVD is the percentage compensated linear vector dipole of the six component moment tensor.

![Fig. 2. Radiation patterns and source parameters computed from synthetic data using the absolute MTI technique. (a) Noise free case. (b) 40% random noise applied to all channels](image)
The percentage of false components is calculated by adding magnitudes of %ISO and %CLVD because for a pure double-couple source, %DC should be $\sim100\%$, and %ISO and %CLVD should both be approximately zero.

It is evident from Fig. 2(a) that the percentage of false components is $\sim8\%$ (this value is not zero because of the less than perfect coverage of the focal sphere that results in a relatively poorly conditioned system of equations). In Fig. 2(b), pseudo-random noise at a level of 40% has been applied to the input data. When this solution is compared with that of the noise-free case, the most obvious change is the blurring of the radiation pattern, indicating an increase in non double-couple components. The percentage of false components increases from $\sim8\%$ to $\sim22\%$. In addition, there is a slight decrease in the accuracy of the fault-plane solutions.

In Fig. 3, a number of different forms of noise are inflicted on the input of the absolute MTI. The results shown are computed using data recorded from a deep-level gold mine in South Africa. Figure 3(a) shows the radiation pattern, fault-plane solutions and source parameters of the control case i.e. the solution computed from seismograms having accurate P- and S-wave picks where no synthetic noise has been applied. The percentage of false components is $\sim6\%$. The radiation pattern of this event becomes slightly blurred when 40% pseudo-random noise is applied to the input.

\begin{figure}[h]
\centering
\begin{subfigure}{0.4\textwidth}
\includegraphics[width=\textwidth]{fig3a.png}
\caption{(a) Control case}
\end{subfigure}\hfill
\begin{subfigure}{0.4\textwidth}
\includegraphics[width=\textwidth]{fig3b.png}
\caption{(b) 40\% random noise applied to all channels}
\end{subfigure}
\vfill
\begin{subfigure}{0.4\textwidth}
\includegraphics[width=\textwidth]{fig3c.png}
\caption{(c) reversed P- and S-wave polarities for site 1}
\end{subfigure}\hfill
\begin{subfigure}{0.4\textwidth}
\includegraphics[width=\textwidth]{fig3d.png}
\caption{(d) bad phase picks for P- and S-wave for site 1, component 1}
\end{subfigure}
\caption{Radiation patterns and source parameters computed from recorded data using the absolute MTI method (a) Control case, (b) 40\% random noise applied to all channels, (c) reversed P- and S-wave polarities for site 1, and, (d) bad phase picks for P- and S-wave for site 1, component 1}
\end{figure}
data (Fig. 3b). In Fig. 3(c), the polarities of the waveforms recorded by the triaxial geophone at site 1 have been reversed, and, Fig. 3(d) shows the effect that incorrect P- and S-wave picks have on the moment tensor solution (for site 1 and component 1). In all cases, except for the control case, the percentage of false components present in the solution increases as a result of the applied noise. This is accompanied by a blurring of the radiation pattern and change in the fault-plane solution.

1.2 Classification of MTI Methods

Numbers of MTI techniques have been proposed since the pioneering paper of Gilbert and Dziewonski (1975). The models of the seismic source and the methods applied differ greatly according to the available data and the purpose of the study. In this study, a distinction is made between the ‘absolute’ and ‘relative’ methods. These broad classes of inversion procedure are based on methods used to estimate the Green’s functions, which describe the wave propagation between the source and receiver.

In the absolute methods, the Green’s functions are evaluated theoretically using an appropriate Earth model and algorithm, or inferred empirically from observations of a known source. One of the difficulties in the absolute inversion (apart from noise affecting the input data) is the accurate estimation of the Green’s functions for geologically complex media. In structurally complex environments, with possible lateral inhomogeneities, it is not always possible to calculate the Green’s functions with adequate accuracy. The result is the introduction of systematic errors into the moment tensor elements. In the mining situation, it is possible to take measurements using geophones placed at depth, thereby removing the effect of surface weathering. However, this advantage is counterbalanced, or possibly exceeded, by the effect of mining voids on the seismic ray-paths.

In contrast to the absolute methods, the relative inversion methods do not require the calculation of theoretical Green’s functions for each event. Relative methods are based on the concept of a common ray-path between a cluster of seismic sources and any receiver, and assume that all the events in the cluster experience the same wave propagation effects to each receiver. Generally, the radiation pattern of a reference event is used to estimate the Green’s functions for events from the same source region (e.g. Patton, 1980; Strelitz, 1980; Oncescu, 1986). In the relative method proposed by Dahm (1996) (known as the relative method without a reference mechanism) the path effects described by the Green’s functions are eliminated analytically – thereby completely avoiding the explicit use of the Green’s functions. However, this method is applicable to clusters of events having different radiation patterns only and, because there is no absolute reference, any of the six components of the moment tensors can be incorrect by an unknown scaling factor. This syndrome can be reduced, but not eliminated, through reference to scalar moment estimates. Furthermore, when the mechanisms are similar the method is extremely sensitive to noise. This is a serious disadvantage in the underground environment where clusters of recorded events might have very similar mechanisms. This problem is accentuated when the method is applied to very small events having similar mechanisms and with signals just above the noise level.
2. Hybrid MTI

A problem common to the absolute and relative MTI methods is their sensitivity to noise or errors in the observations and their dependency on accurate Green’s functions to describe the wave propagation. Factors such as the focusing and defocusing of the ray-paths in structurally complex environments, the degradation of the velocity model due to mining-induced fractures and mining voids on the seismic ray-paths, low signal-to-noise ratios and poor P- and S-wave picks all have adverse effects on the accuracy of the MTI.

The deviations in the ray-paths between a cluster of seismic sources and a particular recording site can result in consistently high or low amplitudes for all the events recorded at that site, resulting in the introduction of systematic errors into the moment tensor elements. The hybrid methods developed by the author are designed to compensate for the various forms of systematic error influencing the waveforms recorded in the underground environment, to enhance signals recorded near a nodal plane in the radiation pattern or to decrease the influence of a low quality observation.

The hybrid methods are in essence weighting schemes that aim to increase the accuracy of the computed moment tensor by diminishing the effect of noisy data on the system of equations. The concept of a weighting scheme is not a new one, and various schemes have been described in the literature (e.g. Udias and Baumann, 1969; Šílený et al., 1992). The method proposed by Udias and Baumann (1969) is perhaps the most relevant to this work. Their weighting scheme was based on the standard deviation of the residuals of the polarisation angles of the S-wave, and on the number of stations with inconsistent P-wave data. These two values were combined into a total error whose minimum was sought. It is an important point to note that the weighting scheme developed by Udias and Baumann was based on the distribution of errors for individual events. The new aspect of the hybrid methods proposed herein is that the correction applied to a particular observation is based on the distribution of residuals (for a particular geophone site, channel and phase) calculated using all of the events in the cluster – this constitutes the relative component of the hybrid methods. The weighting schemes are non-linear and are applied iteratively.

An analogous approach has been applied to the seismic location problem, viz. the so-called ‘JHD method’ (joint hypocentral determination). The relative locations obtained through JHD are usually better than those determined by inversion of more complete and complex velocity models, and the resulting hypocentral locations often give a more focussed picture of the seismicity (e.g. Gibowicz and Kijko, 1994).

2.1 Conceptual Outline of Hybrid MTI Methods

Statistical theory shows that the distributions of the errors of measurement are remarkably regular, i.e. the distributions can be closely approximated by continuous curves referred to as normal error curves (e.g. Press et al., 1990). Using this knowledge, the effect of outliers (data points whose residuals lie ‘far’ from the mean or median error) can be downgraded by the application of the relevant weighting. The errors or residuals indicate the extent of ‘misfit’ of the solution with the measured data, and are the result of a number of error sources.
The weighting factors are computed in three different ways. In Scheme A, the weights are related to the mean of the distribution of residuals. This approach is probably the least robust of all the proposed schemes, since a single large outlying value can dominate the mean (and often does so, owing to the relatively small number of observations or ‘data points’). Because of the problems associated with outliers, Scheme A is merely of academic interest but is included for comparative purposes and to emphasise the need for the schemes that follow. In Scheme B, the weights are related to the median of the distribution of residuals, so that the effect of a solitary outlier is diminished. In Scheme C, a weighting scheme is used which is based on a data point’s distance (measured in standard deviations) from the mean error.

Once the weightings have been applied to the observations, a new set of moment tensor components is computed using the absolute technique. From the new set of moment tensors, a new set of weights is computed. The weights are then applied to the corrected observations, which are subsequently used as input to the absolute MTI as before. (In addition to the weights determined from the residuals, Schemes A and B incorporate a second weighting factor to control the proportion of the weights applied with every iteration.) The process is applied iteratively until a predefined criterion or set of criteria is satisfied. Schemes A, B and C are illustrated conceptually in Fig. 4.

3. Theoretical Description of Hybrid MTI Methods

The goal of all MTI methods is to use observed values of ground displacement to infer properties of the source, as characterised by the moment tensor. By using the representation theorem for seismic sources (Aki and Richards, 1980) and assuming a point source, the displacement field $u_k$ recorded at a receiver $k$ is given by:

$$ u_k(x, t) = G_{ki,j}(x, t; \xi, t') * M_{ij}(\xi, t') $$  \hspace{1cm} (1)

where $G_{ki,j}(x, t; \xi, t')$ are the elastodynamic Green’s functions containing the propagation effects between the source $(\xi, t')$ and receiver. The comma between the indices in Eq. (1) describes the partial derivatives at the source with respect to the coordinates.
after the comma, i.e. \( G_{ki,j} = \frac{\partial G_{ki}}{\partial \xi} \). The \( M_{ij}(\xi, t') \) terms are the nine time-dependent components of the moment tensor.

As the equivalent body forces conserve angular momentum, \( M_{ij} = M_{ji} \), only six of the components are required. Equation (1) can be simplified further by assuming that all of the moment tensor components have the same time-dependency (synchronous source approximation). The temporal part of the moment tensor can then be separated such that

\[
M_{ij}(t') = M_{ij}s(t'),
\]

where \( M_{ij} \) is a set of constant terms, and \( s(t') \) is referred to as the source-time function. Equation (1) becomes:

\[
 u_k(x, t) = \left[ G_{ki,j}(x, t; \xi, t') * s(t') \right] M_{ij}.
\]  

Equation (3) then simplifies to a linear equation:

\[
 u_k = G_{ki,j}M_{ij}.
\]  

Equation (4) can be written in matrix notation as:

\[
 u = Gm
\]

where the vector \( u \) consists of sampled values of the integrated ground displacement or spectral plateaus depending on whether a time or a frequency-domain approach is being taken and is a vector of dimension \( n \). The matrix \( G \) is composed of the Green’s functions in the coordinate system of the receivers and has dimensions \( n \times 6 \). \( m \) is a vector consisting of the moment tensor components \( M_{11}, M_{22}, M_{33}, M_{12}, M_{13} \) and \( M_{23} \). Each component of each station is stacked into the \( u \) vector while the appropriate Green’s function is put into the \( G \) matrix. In most cases \( n \gg 6 \) and the system of equations is, in principle, overdetermined. In order to solve for the components of the moment tensor \( m \), Equation (5) is written as an inverse problem such that:

\[
 m = G^{-1}u,
\]

where \( G^{-1} \) is the generalised inverse of \( G \).

In the weighting schemes that follow, corrections are made to the input data. The corrections are made in stages to maintain stability and to track changes in the moment tensor solutions. Each of the weighting schemes are based on the absolute inversion method described by Eq. (1) to (6). However, it is important to note that in this section the notation is extended slightly, to indicate that the equations are applied to an event (the \( i \)th event) in a cluster (of \( N \) events), rather than to a solitary event. Accordingly, Eq. (5) is rewritten as:

\[
 u_{i}^{obs} = G_{i}m_{i},
\]

where the subscript \( i \) indicates that Eq. (7) applies to the \( i \)th event in a cluster and the superscript \( obs \) indicates that the data contained in the \( u_{i} \) vector are observed values (to distinguish between theoretical values computed later in this section).
Vectors and matrices are denoted using bold letters, and elements of vectors and matrices are given using the same letter but in italics. For example, \( \mathbf{u}_{\text{obs}}^{i} \) is a vector of observed amplitude data for event \( i \) containing \( M \) elements of the form \( u_{ijkl} \) where: the first subscript \( i \) indicates the event number and \( i = (1, 2, 3, \ldots, N) \) where \( N \) is the maximum number of events in the cluster; the second subscript \( j \) denotes the site number where \( j = (1, 2, 3, \ldots, P) \) and \( P \) is the maximum number of geophone or observation sites; the third subscript \( k \) describes the component of the three component sensor and \( k = (1, 2, 3) \); the fourth subscript \( l \) indicates the wave phase such \( l = 1 \) for the P-phase and \( l = 2 \) for the S-phase.

Applying these conventions and considering each equation of the system of equations given by Eq. (7), the displacement of the \( i^{\text{th}} \) event recorded at the \( j^{\text{th}} \) site for geophone component \( k \) and phase observation \( l \) is related to the components of the Green’s functions and moment tensor components by:

\[
\mathbf{u}_{\text{obs}}^{ijkl} = \sum_{r=1}^{6} G_{ijkl} m_{ir}.
\]

### 3.1 Scheme A – Mean Correction

1. Consider a cluster of \( N \) events. The components of the moment tensor \( \mathbf{m}_{i} \) of the \( i^{\text{th}} \) event in the cluster can be computed by writing Eq. (7) as an inverse problem and solving for the vector \( \mathbf{m}_{i} \) such that:

\[
\mathbf{m}_{i} = G_{i}^{-1} \mathbf{u}_{i}^{\text{obs}},
\]

where \( G_{i}^{-1} \) is a \( 6 \times M \) matrix and is the generalised inverse of \( G_{i} \). Since \( M > 6 \) the system of equations is overdetermined and Eq. (9) is solved using the method of least-squares. This approach fits the set of known amplitude data of event \( i \), \( \mathbf{u}_{i}^{\text{obs}} \), to a Green’s function matrix, \( G_{i} \), by computing the most statistically suitable moment tensor \( \mathbf{m}_{i} \) for that event.

2. Once the moment tensors \( \mathbf{m}_{i} \) are computed for each event \( i \) in the cluster, the theoretical displacements are \( \mathbf{u}_{i}^{\text{th}} \) computed using the forward calculation:

\[
\mathbf{u}_{i}^{\text{th}} = G_{i}\mathbf{m}_{i}.
\]

In general, \( \mathbf{u}_{i}^{\text{th}} \neq \mathbf{u}_{i}^{\text{obs}} \) because the system is overdetermined, and the \( \mathbf{m}_{i} \) are the results of the least-squares fit. This fact plays a critical role in the process of hybrid inversion because if \( \mathbf{u}_{i}^{\text{th}} = \mathbf{u}_{i}^{\text{obs}} \) the weighting scheme would have no effect (since the correction would equal zero).

3. A correction or weighting determined by consideration of all the events in the cluster is applied to the input displacements such that:

\[
\mathbf{u}_{ijkl}^{\text{new}} = \mathbf{u}_{ijkl}^{\text{old}} + a_{ijkl} \cdot \mathbf{u}_{ijkl}^{\text{old}},
\]

where \( a_{ijkl} \) is a compound weighting function and \( \mathbf{u}_{ijkl}^{\text{old}} = \mathbf{u}_{ijkl}^{\text{old}} \) for the first iteration only since \( \mathbf{u}_{ijkl}^{\text{old}} \) will be different for each iteration as a result of the correction applied in Eq. (11).
The compound weighting function \( a_{jkl} \) is defined according to:

\[
a_{jkl} = w_{\text{IterNo}} \cdot (\bar{r}_{jkl} - 1).
\]  

(12)

In Eq. (12), \( w_{\text{IterNo}} \) is an attenuation function that ranges between 0 and 1 and controls the amplitude of the applied correction or weighting. The term in brackets \( (\bar{r}_{jkl} - 1) \) is a smoothing function determined from the mean ratio \( \bar{r}_{jkl} \) and observed \( u_{ijkl}^{\text{obs}} \) displacements for the \( j \)th site, \( k \)th component and \( l \)th phase and is computed according to:

\[
\bar{r}_{jkl} = \frac{1}{N_{\text{eq}}} \sum_{i=1}^{N_{\text{eq}}} \left( \frac{u_{ijkl}^{\text{th}}}{u_{ijkl}^{\text{obs}}} \right),
\]  

(13)

where \( N_{\text{eq}} \) is the number of equations and is less than or equal to the number of events \( N \) in the cluster. \( N_{\text{eq}} \) would be less than the total number of events in the cluster if one or more of the geophones were inactive during the occurrence of an event, as is often the case in a mine seismic network. Since the mean ratio \( \bar{r}_{jkl} \) is computed using all the events in the cluster (for \( i = 1 \) to \( N_{\text{eq}} \)) for each site (subscript \( j \)), component (subscript \( k \)) and phase (subscript \( l \)) the \( (\bar{r}_{jkl} - 1) \) term may be thought of as a ‘site correction’.

The motivation for using \( w_{\text{IterNo}} \) is to introduce the correction or weighting gradually in order to gauge the smoothness (or otherwise) of the method’s convergence. The selection of \( w_{\text{IterNo}} \) is subjective and a number of other functions could be used. In this study, \( w_{\text{IterNo}} \) is calculated using:

\[
w_{\text{IterNo}} = \frac{10^{(\text{IterNo}-1)/10}}{10},
\]  

(14)

i.e. \( w_{\text{IterNo}} = \{0.10, 0.12, 0.15, 0.19, 0.25, 0.32, 0.40, 0.50, 0.63, 0.79, 1.0\} \). It is evident from the formulation that the maximum number of iterations is 11 and that the weight of the applied correction increases slowly at the beginning of the iterative procedure, and accelerates during the final stages.

The \( (\bar{r}_{jkl} - 1) \) term of Eq. (12) behaves like a smoothing function. When \( u_{ijkl}^{\text{th}} = u_{ijkl}^{\text{obs}} \) for all events in the cluster (i.e. for \( i = 1 \) to \( N_{\text{eq}} \)), \( \bar{r}_{jkl} \) would equal 1, and as a result, \( a_{jkl} = 0 \). In other words, when the theoretical and observed values of displacement are equal, no correction would be applied. This is hardly ever the case for recorded data and would only occur using synthetic data.

When \( u_{ijkl}^{\text{th}} \approx u_{ijkl}^{\text{obs}} \) for most of the events in the cluster, \( \bar{r}_{jkl} \approx 1 \) resulting in \( a_{jkl} \) being a small value close to zero. In this case, since the observed and theoretical displacements are so similar, the observed data can be considered to have a high degree of accuracy. A very small correction would be applied to the relatively more accurate observed data.

If \( u_{ijkl}^{\text{th}} > u_{ijkl}^{\text{obs}} \) for most of the observed and theoretical displacements, then \( \bar{r}_{jkl} > 1 \) (Eq. (13)). As a result, \( a_{jkl} \) will be positive and the correction will be added to the observation. This assumes \( u_{ijkl}^{\text{th}} > u_{ijkl}^{\text{obs}} \) implies that the observations at the geophones are being underestimated (due to site effects, noise, etc.) and therefore require amplification.

The reverse is true if \( u_{ijkl}^{\text{th}} < u_{ijkl}^{\text{obs}} \) for most of the equations. In this case, \( \bar{r}_{jkl} < 1 \) and \( a_{jkl} \) will be negative. Consequently, the correction will be subtracted from the
observation. It is assumed that $u_{ijkl}^{th} < u_{ijkl}^{obs}$ implies that the observations are being overestimated and therefore require damping.

4. A new set of moment tensors is then recomputed for each event using the corrected data:

$$m_i^{new} = G_i^{-1}u_i^{new}.$$  \(15\)

Return to step 2 and recompute the theoretical displacements using Eq. (10), from which a revised correction can be computed as in steps 3 and 4. Iterating through this sequence, the ‘best’ solution is identified on the basis of the minimum standard error normalised against the scalar moment. The normalisation procedure introduced allows comparisons of the standard errors between iterations. It is necessary because the amplitudes of the linear equations are affected by the weighting scheme, causing changes in the magnitude of the standard error.

Since the mean error ratio contributes to the site correction, this scheme is very sensitive to outliers (outliers are defined as the minimum or maximum values of the ratio described by Eq. (13)). Outliers with large magnitudes will tend to swamp the correction, biasing it towards the less accurate data. For this reason, a Scheme C is developed, where the site correction is based on the median.

### 3.2 Scheme B – Median Correction

This scheme is almost identical to Scheme A, with one exception: Eq. (13) is replaced by:

$$\bar{r}_{jkl} = \text{median} \left( \frac{u_{ijkl}^{th}}{u_{ijkl}^{m}} \right)_{i=1,Neq},$$  \(16\)

where $\bar{r}_{jkl}$ is the median ratio (in contrast to the mean ratio) of the residuals between the theoretical $u^{th}$ and measured $u^{m}$ displacements for the $j$th site, $k$th component and $l$th phase. $Neq$ is the number of equations and is less than or equal to the number of events in the cluster, as before.

In the case of a normal distribution of residuals, the mean and median will be the same, and Scheme B will give the same results as Scheme A. However, if the distribution is skewed, the mean and median residuals will differ and the methods will give different results.

### 3.3 Scheme C – Weighted Mean Correction

While Schemes A and B are very similar, varying only in the definition of $\bar{r}_{jkl}$, Scheme C differs from the previous two schemes because the residuals are computed from the differences between the observed and theoretical displacements (rather than the ratio of theoretical to observed displacement) and the weighting function is relatively simple (in contrast to the complex weighting function that consisted of an attenuation...
function and a smoothing function). As before, the scheme is described in a stepwise fashion.

1. Calculate the theoretical displacements $u_{th}$ from Eq. (10) after solving Eq. (9) in a least-squares sense (as for steps 1 and 2 in Scheme A and B).

2. For the $i^{th}$ event in the cluster, $j^{th}$ site, $k^{th}$ component and $l^{th}$ phase, the residuals $\varepsilon_{ijkl}$ between the theoretical and measured or observed displacements are calculated according to:

$$\varepsilon_{ijkl} = u_{ijkl}^{obs} - u_{ijkl}^{th}. \quad (17)$$

3. A weighting factor $(\omega_{ijkl})$ is determined from the residuals of all the events in the cluster and is applied to the input data in the following way:

$$u_{ijkl}^{new} = \omega_{ijkl} \cdot u_{ijkl}^{old}, \quad (18)$$

where $u_{ijkl}^{old} = u_{ijkl}^{obs}$ for the first iteration and $\omega_{ijkl}$ is computed using:

$$\omega_{ijkl} = \frac{1}{1 + d_{ijkl}^2}. \quad (19)$$

In Eq. (19), $d_{ijkl}$ is the number of standard deviations the residual $\varepsilon_{ijkl}$ is away from the mean residual $\bar{\varepsilon}_{jkl}$ (for event $i$, site $j$, component $k$ and phase $l$) defined by:

$$d_{ijkl} = \frac{\varepsilon_{ijkl} - \bar{\varepsilon}_{jkl}}{s_{jkl}}, \quad (20)$$

where $s_{jkl}$ is the unbiased standard deviation of the residuals. The mean residual $\bar{\varepsilon}_{jkl}$ and unbiased standard deviation $s_{jkl}$ of the residuals are computed using Eq. (21) and Eq. (22), respectively:

$$\bar{\varepsilon}_{jkl} = \frac{1}{N_{eq}} \sum_{i=1}^{N_{eq}} \varepsilon_{ijkl}, \quad (21)$$

$$s_{jkl} = \sqrt{\frac{1}{(N_{eq} - 1)} \sum_{i=1}^{N_{eq}} (\varepsilon_{ijkl} - \bar{\varepsilon}_{jkl})^2}. \quad (22)$$

If $u_{ijkl}^{th} = u_{ijkl}^{obs}$ for all events in the cluster ($i = 1$ to $N_{eq}$) for a particular site (and component and phase), the standard deviation $s_{jkl}$ will be zero, and $\omega_{ijkl} = 1$. (When $s_{jkl} = 0$, Eq. (20) is undefined and some exception handling in the computer code is necessary). Since $\omega_{ijkl} = 1$, no change will be made to the observed data in Eq. (18) (as mentioned before, this will rarely be the case for real, recorded data).

If $u_{ijkl}^{th} \approx u_{ijkl}^{obs}$ for most of the events in the cluster, the standard deviation $s_{jkl}$ will be small, and $\omega_{ijkl}$ will be close to 1 (such that $\omega_{ijkl} < 1$), and very little change will be made to the observed data.

If some of the data are noisy, and $u_{ijkl}^{th} \neq u_{ijkl}^{obs}$ for several events in the cluster, the standard deviation will be larger, and $\omega_{ijkl} \ll 1$. As a result, $\omega_{ijkl}$ will have a damping effect on the noisy data.

4. A new set of moment tensor components is then recomputed for each event using the corrected data:

$$m_{i}^{new} = G_{i}^{-1}u_{i}^{new} \quad (23)$$
5. As with the other schemes, this one can be applied iteratively by returning to step 2 and repeating the process. However, in contrast, the maximum number of iterations is not limited. Once again, the ‘best’ solution is identified on the basis of the minimum standard error normalised against the scalar moment (Eq. (15)).

This scheme shares a common characteristic with Scheme A – a sensitivity to outliers because the correction or weighting is dependent on the standard deviation of the normal distribution.

4. Case Study: Application of Hybrid MTI to Recorded Data

Two of the three hybrid MTI methods (Schemes B and C) were applied to a cluster of events recorded at Oryx Gold Mine because both methods showed similar robustness and stability in the presence of random and systematic noise. However, for the sake of brevity, only the results computed using the hybrid method with a median correction (Scheme B) are presented. The results are compared with those determined using absolute methods.

4.1 Seismic Network and Event Cluster

The seismic network at Oryx Gold Mine is a PRISM system and consists of 7 triaxial geophones located at various depths on the mine’s lease area. (PRISM is a digitally networked real-time seismic monitoring system for mine-wide application developed by

![Plan view of events recorded at Oryx Gold mine](image)
CSIR Miningtek, M & M Systems and Goldfields). The network has a sensitivity of approximately $M_L = -1$ and a location accuracy of around 20 m. The event cluster considered in this study is positioned ahead of an advancing stope at a depth of approximately 2500 m (Fig. 5b). The cluster was selected for closer study due to the noticeably tight spatial grouping, which becomes even more evident when the data set is filtered with respect to event magnitudes. The entire data set recorded between March 1999 and June 2000 consisted of 6495 events and is shown in Fig. 5(a). The filtered data set (displaying events having local magnitudes $>0$) consists of 78 events, and is displayed in Fig. 5(b). Only the seismograms of the filtered data set were available for this study.

### 4.2 Data Description

The cluster selected for investigation consists of 10 of the 78 events, and the local magnitude $M_L$ for these events ranges from 0 to 1.4 (Table 1), recorded over a time period of approximately 6 months.

Not only is the cluster of events tightly grouped in space, but also some of the waveforms of the events are very similar. This implies that the underlying source mechanisms are very similar. The waveform similarity is illustrated in Fig. 6. The waveforms shown are the velocity traces recorded by a single geophone. The waveforms are plotted using the same time scale on the x-axis, velocity on the y-axis, and no filters have been applied. Inspection of the traces reveals that some of the traces are almost indistinguishable from one another.

In many cases, the S-waves are not simple pulses but have an additional higher frequency component. However, this study assumes a simple impulsive source model and, for this reason, the moment tensor solutions will describe the average or dominant source mechanism.

### 4.3 Moment Tensor Inversion

The distances from the event cluster to the receivers range from $\sim 350$ m (site 5) to $\sim 2000$ m (site 3). The Brune source radius of the largest event is $\sim 57$ m ($M_L = 1.4$). Since the source radius of the largest event in this cluster is much less than the distance to the closest geophone site, the far-field approximation is satisfied.

<table>
<thead>
<tr>
<th>Event ID</th>
<th>Date</th>
<th>Time</th>
<th>$M_L$</th>
<th>X (m)</th>
<th>Y (m)</th>
<th>Z (m)</th>
<th>Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>991014004</td>
<td>14/10/1999</td>
<td>08:04:39</td>
<td>0.4</td>
<td>3117570</td>
<td>26493</td>
<td>−2538</td>
<td>13.5</td>
</tr>
<tr>
<td>991118076</td>
<td>18/11/1999</td>
<td>16:50:11</td>
<td>0.5</td>
<td>3117572</td>
<td>26502</td>
<td>−2535</td>
<td>11.8</td>
</tr>
<tr>
<td>991123066</td>
<td>23/11/1999</td>
<td>17:04:10</td>
<td>0.2</td>
<td>3117549</td>
<td>26496</td>
<td>−2502</td>
<td>5.9</td>
</tr>
<tr>
<td>991123074</td>
<td>23/11/1999</td>
<td>17:43:53</td>
<td>1.4</td>
<td>3117571</td>
<td>26532</td>
<td>−2502</td>
<td>6.4</td>
</tr>
<tr>
<td>991127024</td>
<td>27/11/1999</td>
<td>13:35:28</td>
<td>0.9</td>
<td>3117571</td>
<td>26508</td>
<td>−2536</td>
<td>10.7</td>
</tr>
<tr>
<td>991206128</td>
<td>06/12/1999</td>
<td>17:11:46</td>
<td>1.4</td>
<td>3117563</td>
<td>26516</td>
<td>−2511</td>
<td>7.6</td>
</tr>
<tr>
<td>991213000</td>
<td>13/12/1999</td>
<td>04:24:36</td>
<td>0.4</td>
<td>3117573</td>
<td>26525</td>
<td>−2530</td>
<td>7.9</td>
</tr>
<tr>
<td>1000112007</td>
<td>12/01/2000</td>
<td>13:53:23</td>
<td>0.0</td>
<td>3117564</td>
<td>26520</td>
<td>−2528</td>
<td>9.4</td>
</tr>
<tr>
<td>1000328082</td>
<td>28/03/2000</td>
<td>18:56:44</td>
<td>0.5</td>
<td>3117527</td>
<td>26534</td>
<td>−2509</td>
<td>10.6</td>
</tr>
<tr>
<td>1000331112</td>
<td>31/03/2000</td>
<td>23:38:45</td>
<td>0.2</td>
<td>3117524</td>
<td>26525</td>
<td>−2509</td>
<td>15.8</td>
</tr>
</tbody>
</table>
The hybrid MTI method was applied to the event cluster where corrections were applied using medians of errors. Absolute moment tensors were used as starting values for the hybrid methods and were retained for comparative purposes. The inputs to both

Fig. 6. Velocity waveforms plotted using the same time axis as recorded by site 1, component 2. Note similarity of waveforms. P-wave onset and S-wave onset are marked ‘P’ and ‘S’ respectively.
Fig. 7. (a) Typical velocity seismograms. (b) Velocity spectra calculated from the raw traces (solid line), spectral fits (dotted lines) and noise spectra for the P-waves and S-waves for three geophones at one site. Cosine tapering was applied and spectra were smoothed with 3-point averaging.
inversion methods consisted of the spectral plateaus of both P- and S-waves at frequencies below the corner frequency of the time-integrated displacement traces (Fig. 7). The seismograms were not rotated into P, SV and SH phases and Eq. (1) was applied to each phase with good signal to noise ratio, spectral fits and clear polarities. Only phases with well-defined spectral inversions and polarities were used. The spectral fits presented in Fig. (7) were obtained using the method described in Spottswood (1993).

First-motion polarities are not always clear due to several types of departure from the “ideal” data. Noise, head waves and P-wave coda waves prior to S-wave arrivals all complicate the choice of first motion. This is especially true if automatic processing is required. As the spectral plateaus are, in effect, derived from the largest “surge” in ground displacement, the direction of motion of this “surge” was sought. This was achieved by writing a synthetic seismogram based on the fitted spectrum of each phase and by matching this seismogram to the observed seismogram through cross-correlation. The direction of motion was then based on the presence of a single high (positive or negative) peak in the cross-correlation between these two functions. Absence of such a peak indicated that the direction was not well determined.

It is important to note that no outlier rejections were applied. The corrections were based purely on the distribution of the residuals.

To aid interpretation, the computed moment tensors are decomposed into components of source models using the formulation given by Knopoff and Randall (1970). The isotropic component, ISO%, is a measure of the volume change at the source and is calculated from the trace of the Euclidean normalised moment tensor $M$:

$$ISO\% = \frac{100 \text{tr}(M)}{|\text{tr}(M)| + \sum_{i=1}^{3} |m_i^e|},$$

(24)

where the $m_i^e$ are the eigenvalues of the deviatoric moment tensor and are ordered according to magnitude and the trace is computed by summing the eigenvalues $m_i$ of the full tensor: $\text{tr}(M) = m_1 + m_2 + m_3$. The polarity of ISO% gives the direction of motion relative to the source with positive outwards e.g. $-\text{ISO}\%$ indicates an implosion.

The percentage contribution of the double-couple component to the full mechanism can be computed using:

$$DC\% = \frac{m_3^e(1 - 2F)}{|m_3^e(1 - 2F)| + |2m_3^eF|} (100 - \%ISO),$$

(25)

where $F = -m_1^e/m_3^e$ and $m_i^e = m_i - \frac{1}{3}\text{tr}(M)$.

An additional quantity, the deviation $\varepsilon$ of the seismic source from the model of pure double-couple, is calculated. This value is expressed as the ratio of the minimum to maximum deviatoric eigenvalue:

$$\varepsilon = \frac{|m_3^e|}{m_1^e},$$

(26)

where the eigenvalues are ordered in the absolute sense and $\varepsilon = 0$ for a pure double-couple source, and $\varepsilon = 0.5$ for a pure CLVD (Compensated linear vector dipole) (Dziewonski et al., 1981).
4.4 Results

One of the difficulties in moment tensor inversions is proving that the results provide an accurate reflection of reality. The only way this verification can be reliably undertaken is to obtain a detailed fracture mapping of the area of interest and compare the locations and orientations of the computed fault-plane solutions with those of the mapping. Even then, the conclusions often remain speculative. In the absence of such mapping, lesser techniques must be used. In this particular case study, the similarity of the waveforms plays an important role in assessing the integrity of the hybrid moment tensor solutions in the sense of whether or not they are any better than those determined using absolute methods. Since the input waveforms of the cluster show such close similarity, it is reasonable to expect the output moment tensors and their corresponding radiation patterns to exhibit similar likenesses.

The radiation patterns and fault-plane solutions of the cluster computed using the absolute MTI method are shown in Fig. 8. The Cartesian moment tensors, source

Table 2. Cartesian moment tensor components for the Oryx cluster (Absolute MTI method)

<table>
<thead>
<tr>
<th>Event ID</th>
<th>Cartesian moment tensors (10^9 N.m) rotated into geographical (North, East, Down) system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M_{11}</td>
</tr>
<tr>
<td>991014004</td>
<td>−3.00</td>
</tr>
<tr>
<td>991118076</td>
<td>−5.00</td>
</tr>
<tr>
<td>991123066</td>
<td>−3.46</td>
</tr>
<tr>
<td>991123074</td>
<td>−9.76</td>
</tr>
<tr>
<td>991127024</td>
<td>−12.65</td>
</tr>
<tr>
<td>991206128</td>
<td>−9.32</td>
</tr>
<tr>
<td>991213000</td>
<td>−2.49</td>
</tr>
<tr>
<td>1000112007</td>
<td>0.53</td>
</tr>
<tr>
<td>1000328082</td>
<td>4.58</td>
</tr>
<tr>
<td>1000331112</td>
<td>−0.84</td>
</tr>
</tbody>
</table>
parameters and fault-plane solutions for the cluster are listed in Tables 2, 3 and 4. It is evident from the radiation patterns (Fig. 8) and their corresponding fault-plane solutions that the results computed using the absolute MTI are more varied than would

**Table 3.** Source parameters for the Oryx cluster (Absolute MTI method)

<table>
<thead>
<tr>
<th>Event ID</th>
<th>Deviation from DC (ε)</th>
<th>ISO%</th>
<th>DC%</th>
</tr>
</thead>
<tbody>
<tr>
<td>991014004</td>
<td>0.58</td>
<td>−20.0</td>
<td>12.4</td>
</tr>
<tr>
<td>991118076</td>
<td>0.66</td>
<td>−19.2</td>
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<tr>
<td>991123066</td>
<td>0.33</td>
<td>−25.1</td>
<td>34.2</td>
</tr>
<tr>
<td>991123074</td>
<td>0.43</td>
<td>−14.5</td>
<td>68.5</td>
</tr>
<tr>
<td>991127024</td>
<td>0.55</td>
<td>−10.5</td>
<td>65.7</td>
</tr>
<tr>
<td>991206128</td>
<td>0.28</td>
<td>−1.00</td>
<td>43.4</td>
</tr>
<tr>
<td>991213000</td>
<td>0.02</td>
<td>1.71</td>
<td>95.4</td>
</tr>
<tr>
<td>1000112007</td>
<td>0.45</td>
<td>18.9</td>
<td>67.8</td>
</tr>
<tr>
<td>1000328082</td>
<td>0.36</td>
<td>33.3</td>
<td>70.4</td>
</tr>
<tr>
<td>1000331112</td>
<td>0.19</td>
<td>0.8</td>
<td>62.7</td>
</tr>
</tbody>
</table>

**Table 4.** Fault-plane solutions for the Oryx cluster (Absolute MTI method)

<table>
<thead>
<tr>
<th>Event ID</th>
<th>Fault-plane solution 1 (degrees)</th>
<th>Fault-plane solution 2 (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strike</td>
<td>Dip</td>
</tr>
<tr>
<td>991014004</td>
<td>194.9</td>
<td>18.9</td>
</tr>
<tr>
<td>991118076</td>
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<tr>
<td>991123074</td>
<td>270.4</td>
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<tr>
<td>991127024</td>
<td>161.0</td>
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<td>991206128</td>
<td>337.6</td>
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<td>17.3</td>
</tr>
<tr>
<td>1000328082</td>
<td>245.0</td>
<td>55.9</td>
</tr>
<tr>
<td>1000331112</td>
<td>152.0</td>
<td>12.6</td>
</tr>
</tbody>
</table>

**Fig. 9.** Radiation patterns and fault-plane solutions for the cluster computed with the hybrid MTI using a median correction (Scheme B)
be expected from a source cluster having very similar waveforms. In Table 3, ISO% and DC% do not add up to 100% because ISO% is computed from the complete moment tensor whereas DC% is computed from the deviatoric part of the moment tensor.

The hybrid MTI method using a median correction gives radiation patterns and fault-plane solutions showing a high degree of similarity (Fig. 9), indicating that these solutions are probably more accurate reflections of reality than the absolute moment tensor solutions. The Cartesian moment tensor components, source parameters and fault-plane solutions for the cluster are listed in Tables 5, 6 and 7.

There is a noticeable decrease in the isotropic components (compare ISO% of Tables 3 and 6) in this second set of results. There has been much discussion in the literature about the interpretation of non-double-couple components (i.e. the presence of isotropic components and deviations from the pure double-couple model). Most of the debate is centred on whether the non-double-couple components are intrinsic to the source rupture process or artefacts of processing (due to inappropriate modelling of the ray-path, the presence of noise, invalid source models, etc.). This study shows that the percentage of isotropic component can be decreased significantly by application of the hybrid MTI method with median correction.

Table 5. Cartesian moment tensor components for the Oryx cluster (Hybrid MTI)

<table>
<thead>
<tr>
<th>Event ID</th>
<th>Cartesian moment tensors (10^9 N.m) rotated into geographical (North, East, Down) system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{11}$</td>
</tr>
<tr>
<td>991014004</td>
<td>-2.84</td>
</tr>
<tr>
<td>991118076</td>
<td>-3.77</td>
</tr>
<tr>
<td>991123066</td>
<td>-1.09</td>
</tr>
<tr>
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<td>-14.72</td>
</tr>
<tr>
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<td>-7.91</td>
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<td>1000331112</td>
<td>-5.26</td>
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Table 6. Source parameters for the Oryx cluster (Hybrid MTI)

<table>
<thead>
<tr>
<th>Event ID</th>
<th>Deviation from DC (ε)</th>
<th>ISO%</th>
<th>DC%</th>
</tr>
</thead>
<tbody>
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<td>991014004</td>
<td>0.03</td>
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<tr>
<td>991118076</td>
<td>0.19</td>
<td>-3.5</td>
<td>62.3</td>
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<tr>
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<td>0.23</td>
<td>-5.0</td>
<td>76.1</td>
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<td>991123074</td>
<td>0.11</td>
<td>1.8</td>
<td>77.5</td>
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<td>991127024</td>
<td>0.09</td>
<td>2.4</td>
<td>82.31</td>
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<td>991206128</td>
<td>0.33</td>
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<td>46.8</td>
</tr>
<tr>
<td>1000331112</td>
<td>0.31</td>
<td>16.2</td>
<td>72.1</td>
</tr>
</tbody>
</table>
The ‘sharpening up’ of the solutions described earlier is an encouraging phenomenon, and is a strong indicator of the hybrid method’s potential for use as a standard processing tool for mine seismicity.

5. Practical Implications of this Study

As already alluded to in the summary, insight into the source mechanisms of mine tremors contributes to understanding the various modes of failure observed around underground excavations in the mining environment. The planes of failure may be caused by stresses induced by mining or be of geological origin. Knowledge of the geometry of these failure planes is critical in the mine-planning process so that tunnels are not planned in unfavourable orientations with respect to major geological structures.

The MTI methods currently applied on the mines are not always reliable and are sensitive to many different sources of error. The hybrid MTI methods developed by the author aim to increase the accuracy of the moment tensor solutions by reducing the effect of noisy data and correcting for geophone site effects. In the case study, the solutions computed using the hybrid method are more focussed than those calculated using the absolute method and show fine distinctions amongst the events not evident from the absolute solutions. The improved moment tensor solutions will then provide more opportunities for interpreting mine seismicity in terms of driving stresses and geological features, ultimately leading to improved safety underground.

A computer program, the ‘MTI toolbox’, was written by the author to perform the hybrid MTI techniques. This program is compatible with AURA and enables moment tensors to be computed quickly and easily. (AURA is the seismological processing and analysis software written by CSIR Miningtek that reads data recorded by both PRISM and GMM systems.) Since the analysis in the case study was based on these automatic procedures, the hybrid MTI methods have the potential of being applied routinely in processing mine seismic data.

The hybrid MTI methods are being used in the following environments:

- The research environment. The hybrid MTI methods were applied extensively to seismograms recorded by an ISS seismic system and converted into PRISM format.

### Table 7. Fault-plane solutions for the Oryx cluster (Hybrid MTI)

<table>
<thead>
<tr>
<th>Event ID</th>
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<th>Dip</th>
<th>Rake</th>
<th>Strike</th>
<th>Dip</th>
<th>Rake</th>
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<tr>
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<td>134.6</td>
<td>10.3</td>
<td>179.8</td>
<td>224.7</td>
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<td>991123066</td>
<td>142.4</td>
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<td>87.7</td>
<td>-82.0</td>
</tr>
<tr>
<td>991123074</td>
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<td>39.8</td>
<td>88.1</td>
<td>-81.2</td>
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for processing in AURA during the course of a SIMRAC project (www.simrac.co.za) that investigated the mechanisms of failure near a dip-stabilizing pillar (GAP604). The moment tensor analysis was useful in determining failure trends and assessing the regional stress state of the study area.

- The hybrid MTI methods have been applied to seismograms recorded by a PRISM system installed at Oryx Gold Mine, South Africa. The hybrid MTI software performs the calculations on a data file generated by AURA.
- Application of the hybrid MTI method is not limited to the mining environment but may also be useful in the laboratory. In rock mechanics, studies of acoustic emission (AE) data have often been used to monitor damage and failure in brittle materials. MTI methods applied to AE data are subject to similar sources of error as seismic data recorded on a macro-scale in the mining environment. Since these micro-events also occur in clusters and the common ray-path approximation holds, the hybrid MTI methods could be used to enhance the moment tensor solutions of acoustic emission data, providing greater insight into the brittle fracture process. The hybrid MTI methods have been applied to AE data recorded at CSIR Miningtek. A series of uniaxial compression tests were performed on 96-mm-diameter quartzite samples to understand the scaling of rock fracture processes. The recorded AEs were compared to mining-induced seismicity and natural earthquakes. The moment tensors were found to have double-couple components, indicating that shearing was occurring. The work is described in detail in Sellers et al. (2003).
- The hybrid MTI method has been applied by Finck et al. (2003) to AE data recorded during a splitting test of a concrete cube. This work contributes towards a collaborative project entitled ‘Non-destructive evaluation of concrete structures using acoustic and electro-magnetic methods’ funded by the German National Science Foundation (FOR 384).

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References


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